

# Cal Poly Department of Mathematics

## Puzzle of the Week

October 27 - November 2, 2011

Suggested by Kent Morrison:

Determine the number of factors of 2 in the number  $(2^n)!$ . If possible give an explicit answer in terms of  $n$ .

*Solutions should be submitted to Morgan Sherman:*

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*before next Friday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.*

<http://www.calpoly.edu/~sherman1/puzzleoftheweek>

*Solution:* There are  $2^n - 1$  factors of 2 in  $2^n!$ .

Each of the even numbers contribute one factor of two, and there are  $\frac{1}{2} \cdot 2^n$  of these. Of these half contribute another factor of two, and of those another half contribute another factor, etc. So in total we have

$$\begin{aligned} \frac{1}{2} \cdot 2^n + \frac{1}{4} \cdot 2^n + \dots + \frac{1}{2^n} \cdot 2^n &= 2^{n-1} + 2^{n-2} + \dots + 2 + 1 \\ &= \frac{2^n - 1}{2 - 1} = 2^n - 1 \end{aligned}$$

factors of two.