

Cal Poly Department of Mathematics

Puzzle of the Week

September 22-28, 2011

Find a multiple of 2011 which has the (decimal) form $\underbrace{111 \dots 1}_k$, or show no such multiple exists.

Solutions should be submitted to Morgan Sherman:

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before next Friday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

<http://www.calpoly.edu/~sherman1/puzzleoftheweek>

Solution: The number $\underbrace{111 \dots 1}_{2010 \text{ digits}}$ is such a multiple of 2011 (there are others as well).

Here's one way to see this with some Number Theory. Using "Fermat's Little Theorem" we know that $a^{p-1} \equiv 1 \pmod{p}$ when p is a prime not dividing a . Now it happens that 2011 is a prime hence

$$10^{2011-1} = m \cdot 2011 + 1$$

for some integer m . Since $10^{2010} - 1 = \underbrace{999 \dots 9}_{2010 \text{ digits}}$ we get the above result after dividing both sides by 9.