Cal Poly Department of Mathematics

Puzzle of the Week

May 6-12, 2011

From Tom O'Neil:

The sides of a triangle have lengths which form 3 consecutive integers and the largest angle is twice the smallest angle. What is the cosine of the smallest angle?

Solutions should be submitted to Morgan Sherman:

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before next Friday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

http://www.calpoly.edu/~sherman1/puzzleoftheweek

Solution: The cosine of the smallest angle is $\frac{3}{4}$.

Let θ denote the smallest angle. Labeling the sides to have lengths of $n-1, n, n+1$ we see that the side opposite θ must have length $n-1$ while the side opposite 2θ must have length $n+1$. By the Law of Sines:

$$\frac{\sin\theta}{n-1} = \frac{\sin 2\theta}{n+1} = \frac{2\sin\theta\cos\theta}{n+1} \implies \cos\theta = \frac{n+1}{2(n-1)}.$$

On the other hand, by the Law of Cosines we have

$$(n-1)^2 = n^2 + (n+1)^2 - 2n(n+1)\cos\theta \implies \cos\theta = \frac{n+4}{2(n+1)}.$$

Equating these two expressions and simplifying we arrive at $n = 5$. So find that we actually have a triangle with lengths 4, 5, and 6 and

$$\cos\theta = \frac{5+1}{2(5-1)} = \frac{3}{4}$$

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