

# Cal Poly Department of Mathematics

## Puzzle of the Week

Apr 29-May 5, 2011

From Tom O'Neil:

What are the possible values for  $n$  if 2010 can be written as the sum of  $n$  consecutive positive integers?

*Solutions should be submitted to Morgan Sherman:*

*Dept. of Mathematics, Cal Poly  
Email: sherman1 -AT- calpoly.edu  
Office: bldg 25 room 310*

*before next Friday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.*

<http://www.calpoly.edu/~sherman1/puzzleoftheweek>

*Solution:* The possible values for  $n$  are 1, 3, 4, 5, 12, 15, 20, and 60.

If 2010 is the sum of  $n$  consecutive integers beginning with  $k > 0$  then:

$$2010 = k + (k + 1) + \dots + (k + n - 1) = \sum_{i=k}^{n+k-1} i - \sum_{j=1}^{k-1} j = \frac{(n+k-1)(n+k)}{2} - \frac{(k-1)k}{2}$$

which can be re-written as

$$n(2k + n - 1) = 4020 = 2^2 \cdot 3 \cdot 5 \cdot 67.$$

We see that  $n$  cannot be larger than  $\sqrt{4020} \approx 63$ . Also notice that exactly one of  $n$  and  $2k + n - 1$  is even, the other will be odd. So if  $n$  is even it must be a multiple of 4; if  $n$  is odd it can only be produced with the factors 1, 3, and 5. So the possibilities are:

$$n = 1, 3, 5, 15(\text{odd}); \quad n = 4, 12, 20, 60(\text{even}).$$

For example, if  $n = 15$  then  $k = 127$  and

$$2010 = 127 + 128 + 129 + \dots + 141.$$