Cal Poly Department of Mathematics

Puzzle of the Week

Feb 11-17, 2011

From Vince Bonini:

For an arbitrary positive integer n calculate

$$\int_{-\infty}^{\infty} \frac{dx}{(x^{2n}+1)(x^2+1)}$$

Hint: A previous Puzzle of the Week may prove helpful...
Note: Originally the exponent on the first x in the denominator was allowed to be odd, but for those values the denominator has a zero and the integral diverges

Solutions should be submitted to Morgan Sherman:

Dept. of Mathematics, Cal Poly
Email: sherman1 -AT- calpoly.edu
Office: bldg 25 room 310

before next Friday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

http://www.calpoly.edu/~sherman1/puzzleoftheweek

Solution: The integral converges to $\pi/2$.

One can make the trigonometric substitution $x = \tan\theta$ and then follow the argument from Puzzle of the Week # 40. However I prefer the more direct method that Lawrence Sze (and others) submitted: if I denotes the integral in question then with the substitution $u = 1/x$ we find

$$\frac{1}{2}I = \int_0^{\infty} \frac{dx}{(x^{2n}+1)(x^2+1)} = \int_{\infty}^{0} \frac{-du/u^2}{(\frac{1}{u^{2n}}+1)(\frac{1}{u^2}+1)} = \int_0^{\infty} \frac{u^{2n}du}{(u^{2n}+1)(u^2+1)}$$

and therefore

$$I = \frac{1}{2}I + \frac{1}{2}I = \int_0^{\infty} \frac{dx}{(x^{2n}+1)(x^2+1)} + \int_0^{\infty} \frac{x^{2n}dx}{(x^{2n}+1)(x^2+1)} = \int_0^{\infty} \frac{dx}{x^2+1} = \frac{\pi}{2}.$$