

Cal Poly Department of Mathematics

Puzzle of the Week

Jan 14-20, 2011

From Jeremy Kun:

Is it possible to delete one of $1!, 2!, 3!, \dots, 100!$ so that the product of the remaining terms is a perfect square?

Solutions should be submitted to Morgan Sherman:

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before next Friday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

<http://www.calpoly.edu/~sherman1/puzzleoftheweek>

Solution: By removing $50!$ the remaining product is a perfect square.

We compute:

$$\begin{aligned} 1! \cdot 2! \cdot 3! \cdot \dots \cdot 100! &= (1!)^2 2 \cdot (3!)^2 4 \cdot (5!)^2 6 \cdot \dots \cdot (99!)^2 100 \\ &= (2 \cdot 4 \cdot 6 \cdot \dots \cdot 100) (1!)^2 \cdot (3!)^2 \cdot (5!)^2 \cdot \dots \cdot (99!)^2 \\ &= 50! \cdot 2^{50} \cdot (1!)^2 \cdot (3!)^2 \cdot (5!)^2 \cdot \dots \cdot (99!)^2 \\ &= 50! \cdot (\text{perfect square}) \end{aligned}$$