

Cal Poly Department of Mathematics

Puzzle of the Week

Jan 7-13, 2011

From Jeff Liese:

How many ways can you arrange the number $1, 2, 3, \dots, 2011$ so that each number after the first is within one of a number somewhere to its left? [For example, if 17 is the 2nd number then the first must be 16 or 18.]

Solutions should be submitted to Morgan Sherman:

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before next Friday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

<http://www.calpoly.edu/~sherman1/puzzleoftheweek>

Solution: The number of arrangements is 2^{2010} .

I am going to plagiarize Lawrence Sze's solution, which I thought was especially nice (and I had not seen before): Given such an arrangement every term after the first is either one **A**bove or one **B**elow the range of numbers to its left, but not both as can be seen by reading from left to right. Thus each legal arrangement corresponds to a sequence of 2010 **A**s and **B**s. Moreover every such sequence corresponds to a legal arrangement by taking the first term to be $1 +$ the number of **B**s, then filling in the remaining terms appropriately according to their letter.