

Cal Poly Department of Mathematics

Puzzle of the Week

Oct 29 - Nov 4, 2010

A right circular cone with base radius 1 and height 5 has a cube inscribed in it, with one face of the cube contained in the base of the cone. Find the side length of the cube.

Solutions should be submitted to Morgan Sherman:

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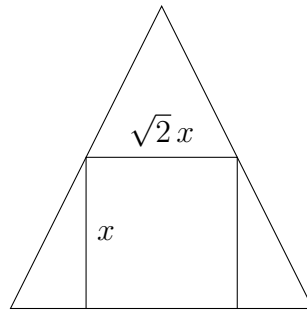
Office: bldg 25 room 310

before next Friday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

<http://www.calpoly.edu/~sherman1/puzzleoftheweek>

The cube has side length $\frac{10}{5\sqrt{2} + 2}$.

This problem (with a slight modification) was problem A1 on the 1998 Putnam Exam. Let the side length of the cube be x . Viewing the cone from the side we can find the following diagram:



Here the upper horizontal line is a diagonal across the top of the cube, and the large triangle has height 5 and base 2. From similar triangles we see $\frac{5-x}{5} = \frac{\sqrt{2}x}{2}$. Solving this equation for x yields the answer above.