Calculate, with justification, the integral
\[ \int_{2}^{3} \frac{\sin \sqrt{13 - x}}{\sin \sqrt{13 - x} + \sin \sqrt{8 + x}} \, dx \]

_Solutions should be submitted to Morgan Sherman:_

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_before next Friday. Those with correct and complete solutions will have their names listed on the puzzle’s web site (see below) as well as in next week’s email announcement. Anybody is welcome to make a submission.

http://www.calpoly.edu/~sherman1/puzzleoftheweek

_Solution:_ The value of the integral is 1/2.

This problem is taken from (with modifications) problem B1 of the 1987 Putnam Exam. The trick here is to use some symmetry: the change of variables \( u = 5 - x \) transforms \( 13 - x \) into \( 8 + u \) and vice-versa. At the same time as \( x \) ranges over \([2, 3]\) so does \( u \). Hence we try the substitution:

\[
I = \int_{2}^{3} \frac{\sin \sqrt{13 - x}}{\sin \sqrt{13 - x} + \sin \sqrt{8 + x}} \, dx
\]

\[
= \int_{3}^{2} \frac{\sin \sqrt{8 + u}}{\sin \sqrt{13 - u} + \sin \sqrt{8 + u}} \, du
\]

\[
= \int_{2}^{3} \frac{\sin \sqrt{8 + x}}{\sin \sqrt{13 - x} + \sin \sqrt{8 + x}} \, dx
\]

and therefore

\[
I + I = \int_{2}^{3} \frac{\sin \sqrt{13 - x} + \sin \sqrt{8 + x}}{\sin \sqrt{13 - x} + \sin \sqrt{8 + x}} \, dx = \int_{2}^{3} \, dx = 1.
\]

Hence the answer above.