Cal Poly Department of Mathematics

Puzzle of the Week

Oct 22 - 28, 2010

From Tom O'Neil:

Let C be a circle of radius r and center O. Fix a point Q different from O. Find the locus ℓ of the centroids of the triangles OPQ as P moves about the circumference of the circle and show that the area enclosed by ℓ is independent of Q.

Solutions should be submitted to Morgan Sherman:

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before next Friday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

http://www.calpoly.edu/~sherman1/puzzleoftheweek

Solution:

The locus is a circle of radius $\frac{r}{3}$ centered at the point $\frac{1}{3}$rd the distance from O to Q. Therefore its area is independent of Q.

Let $\mathbf{u} = \vec{OQ}$ and $\mathbf{v} = \vec{OP}$. One can check that the centroid of the triangle OPQ is given by $\frac{1}{3}\mathbf{u} + \frac{1}{3}\mathbf{v}$. Now setting $\mathbf{v} = (r\cos t, r\sin t)$ we see that the locus is described by

$$\mathbf{r}(t) = \frac{1}{3}\mathbf{u} + \frac{r}{3}(\cos t, \sin t), \quad 0 \le t \le 2\pi,$$

hence the answer above.