

Cal Poly Department of Mathematics

Puzzle of the Week

Oct 8 - 14, 2010

From Tom O'Neil, and a follow-up to last week's problem:

In an urn there are R red balls and B blue balls. As it turns out if two balls are drawn simultaneously (and randomly) the likelihood that they are the same color is 50%. Characterize all possible values for R and B .

Solutions should be submitted to Morgan Sherman:

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before next Friday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

<http://www.calpoly.edu/~sherman1/puzzleoftheweek>

Solution:

R and B must be consecutive *triangular numbers* (i.e. those of the form $\binom{n}{2} = \frac{n(n-1)}{2}$).

To see this note that the condition in the problem means that

$$\frac{\binom{R}{2} + \binom{B}{2}}{\binom{R+B}{2}} = \frac{1}{2}$$

which, after some algebra, simplifies to $(R - B)^2 = R + B$. Now writing $R - B = n$ and $R + B = n^2$, where n is an arbitrary integer, we see that R and B have the form $R = \binom{n+1}{2}$ and $B = \binom{n}{2}$. Hence the solution above.