

# Cal Poly Department of Mathematics

## Puzzle of the Week

Sep 24 - 30, 2010

Let  $m, b$  be positive real numbers. The line  $y = mx + b$  intersects the parabola  $y = x^2$  in two points, say  $A$  and  $B$ . Find the point  $P$  on the parabola, lying between  $A$  and  $B$ , such that the triangle  $ABP$  has maximal area.

*Solutions should be submitted to Morgan Sherman:*

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*before next Friday. Those with correct and complete solutions will have their names listed in next week's email announcement. Anybody is welcome to make a submission.*

### Solution:

The point has coordinates  $(\frac{m}{2}, \frac{m^2}{4})$ . Here is a quick argument to find the solution. Let  $\ell$  be the line  $y = mx + b$ . By taking the base of the triangle to be the line segment  $AB$  we see that to maximize the area we need to find the point  $P$  which is furthest from the line  $\ell$ . Suppose  $P = (x, x^2)$  is this point and consider the line  $\ell'$  passing through  $P$  and parallel to  $\ell$ . Then  $\ell'$  must be tangent to the parabola, otherwise  $P$  would not be the furthest point from  $\ell$ . Therefore  $2x = m$  and we find the solution above.