Cal Poly Department of Mathematics

Puzzle of the Week

May 14-20, 2010

From Kent Morrison:
After grading an exam I must enter the scores on my grade sheet which has students listed alphabetically. While going through the stack of exams it occasionally happens that two consecutive exams are also listed consecutively on the grade sheet (in the same order). For a class of 35 students what is the expected number of times this will happen for one set of exam papers?

Solutions should be submitted to Morgan Sherman:

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before next Friday. Those with correct and complete solutions will have their names listed in next week's email announcement. Anybody is welcome to make a submission.

Solution:

The expected number is $\frac{34}{35} \approx 0.97$.

Consider the 34 consecutive pairs of positions: $[1,2], [2,3], [3,4], \ldots, [34,35]$. For the exams in any one of these pairs to be in consecutive alphabetical order the first exam must be one of the first 34 in alphabetical order, and the second exam must be exactly the one which follows. This happens with probability $\frac{34}{35} \times \frac{1}{34} = \frac{1}{35}$. Since there are 34 of these pairs we expect the total number to be $34 \times \frac{1}{35}$, hence the solution above.