Cal Poly Department of Mathematics

Puzzle of the Week

May 7-13, 2010

Calculuate:

$$\prod_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right)$$

Solutions should be submitted to Morgan Sherman:

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before next Friday. Those with correct and complete solutions will have their names listed in next week's email announcement. Anybody is welcome to make a submission.

Solution: The product converges to $\frac{1}{2}$.

Since $1-\frac{1}{n^2}=\frac{n^2-1}{n^2}=\frac{(n-1)(n+1)}{n^2}$ we see that

$$\begin{aligned}\prod_{n=2}^{N}\left(1-\frac{1}{n^2}\right) &= \prod_{n=2}^{N}\left(\frac{(n-1)(n+1)}{n^2}\right)\\ &= \frac{(1\cdot 2\cdot 3\cdot\ldots\cdot(N-1))(3\cdot 4\cdot 5\cdot\ldots\cdot(N+1))}{(1\cdot 2\cdot 3\cdot\ldots\cdot N)(1\cdot 2\cdot 3\cdot\ldots\cdot N)}\\ &= \frac{N+1}{(N)(2)}\\ &\to \frac{1}{2}\text{ as } N\to\infty\end{aligned}$$