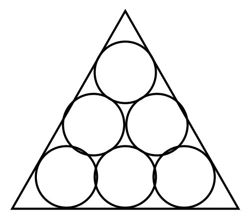
Cal Poly Department of Mathematics

Puzzle of the Week

April 30-May 6, 2010

Circles of equal radii are packed into an equilateral triangle with unit area in such a way that n circles are adjacent along the bottom edge, $n-1$ directly above them, $n-2$ above those, etc. (see picture, with $n=3$). If A_n is the sum of the areas of the circles find $\lim_{n\to\infty} A_n$.



Solutions should be submitted to Morgan Sherman:

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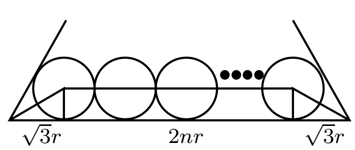
before next Friday. Those with correct and complete solutions will have their names listed in next week's email announcement. Anybody is welcome to make a submission.

<u>*Solution:*</u> The limiting value of A_n is $\dfrac{\pi}{2\sqrt{3}}$

From the following diagram



we can compute the side length of the triangle as $L = 2(n+\sqrt{3})r$, where r is the radius of one circle. On the other hand using Heron's formula (say) we find that L must satisfy $1 = \sqrt{\frac{3L}{2}(\frac{L}{2})^3}$. Thus $2(n+\sqrt{3})r = \frac{2}{\sqrt[4]{3}}$ and therefore $r = 1/(\sqrt[4]{3}(n+\sqrt{3}))$. Now there are a total of $1+2+3+\ldots+n = n(n+1)/2$ circles in the triangle so we see

$$A_n = \frac{n(n+1)}{2}\pi r^2 = \frac{\pi n(n+1)}{2\sqrt{3}(n+\sqrt{3})^2} \to \frac{\pi}{2\sqrt{3}} \text{ as } n\to\infty$$