

# Cal Poly Department of Mathematics

## Puzzle of the Week

April 23-29, 2010

How many times, in one 24-hour day, does the minute hand and the hour hand on an analog clock make a right angle?

*Solutions should be submitted to Morgan Sherman:*

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*before next Friday. Those with correct and complete solutions will have their names listed in next week's email announcement. Anybody is welcome to make a submission.*

*Solution:* There are 44 times during the day that the hour hand and the minute hand are at right angles.

Viewing the hour hand as a vector rotating about the origin we can write it down as a vector-valued function of time  $t$  – measured in days – as

$$\mathbf{h}(t) = \langle \sin(2t \cdot 2\pi), \cos(2t \cdot 2\pi) \rangle$$

while the minute hand is given by

$$\mathbf{m}(t) = \langle \sin(24t \cdot 2\pi), \cos(24t \cdot 2\pi) \rangle.$$

Then the number of times the minute hand and the hour hand make a right angle during the day is the number of zeroes of the function

$$f(t) = \mathbf{h}(t) \cdot \mathbf{m}(t) = \sin(2t \cdot 2\pi) \sin(24t \cdot 2\pi) + \cos(2t \cdot 2\pi) \cos(24t \cdot 2\pi) = \cos((24-2)t \cdot 2\pi) = \cos(22t \cdot 2\pi)$$

This function has precisely 22 periods as  $t$  ranges from 0 to 1 and each period contributes two zeroes, hence the solution above.