Cal Poly Department of Mathematics

Puzzle of the Week

April 2-8, 2010

1000 light bulbs are connected to 1000 switches, all placed in a very long row, all initially switched off. Person 1 goes and flips the switch on every light, then person 2 flips the switch on every second light, then person 3 flips the switch on every 3rd light, and so on.

After person 1000 finishes how many bulbs are switched on?

Solutions should be submitted to Morgan Sherman:

Dept. of Mathematics, Cal Poly
Email: sherman1 -AT- calpoly.edu
Office: bldg 25 room 310

before next Friday. Those with correct and complete solutions will have their names listed in next week's email announcement. Anybody is welcome to make a submission.

Solution: There are 31 light bulbs left on.

Note that light bulb N will have its switch flipped once for each positive divisor of N. Since all the bulbs are initially off only those numbers with an odd number of positive divisors will be left on. Now if k is a divisor of N then so is N/k and these will always be distinct divisors unless $N = k^2$. Therefore we can group the divisors of any integer into pairs, except for the perfect squares $N = k^2$ where the divisor k will not have a partner. This tells us that an integer has an odd number of positive divisors if and only if it is a perfect square. Since there are exactly 31 perfect squares less than 1000 ($31^2 = 961,\ 32^2 = 1024$) we get the answer above.