1000 light bulbs are connected to 1000 switches, all placed in a very long row, all initially switched off. Person 1 goes and flips the switch on every light, then person 2 flips the switch on every second light, then person 3 flips the switch on every 3rd light, and so on.

After person 1000 finishes how many bulbs are switched on?

Solutions should be submitted to Morgan Sherman:
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before next Friday. Those with correct and complete solutions will have their names listed in next week’s email announcement. Anybody is welcome to make a submission.

Solution: There are 31 light bulbs left on.

Note that light bulb $N$ will have its switch flipped once for each positive divisor of $N$. Since all the bulbs are initially off only those numbers with an odd number of positive divisors will be left on. Now if $k$ is a divisor of $N$ then so is $N/k$ and these will always be distinct divisors unless $N = k^2$. Therefore we can group the divisors of any integer into pairs, except for the perfect squares $N = k^2$ where the divisor $k$ will not have a partner. This tells us that an integer has an odd number of positive divisors if and only if it is a perfect square. Since there are exactly 31 perfect squares less than 1000 ($31^2 = 961, 32^2 = 1024$) we get the answer above.