

Cal Poly Department of Mathematics

Puzzle of the Week

Feb 12-18, 2010

Here's another puzzle involving three-digit numbers: For every three-digit number $n = ABC$ define

$$S(n) = A + B + C + (A \times B) + (A \times C) + (B \times C) + (A \times B \times C)$$

Find all (three-digit) numbers n with $S(n) = n$.

Solutions should be submitted to Morgan Sherman:

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before next Friday. Those with correct and complete solutions will have their names listed in next week's email announcement. Anybody is welcome to make a submission.

Solution:

The number n must be of the form (in decimal notation) $x99$ for some digit x .

I borrowed this puzzle from Brian Miceli – a friend of our own Jeff Liese. Writing n in decimal notation as “ ABC ” for digits A, B, C we see that the equation $n = S(n)$ can be written as

$$100A + 10B + C = A + B + C + AB + AC + CB + ABC$$

which simplifies to

$$99A + 9B = AB + AC + BC + ABC$$

or, more suggestively,

$$99A + 9B = (BC + B + C)A + (C)B.$$

Since $B, C \leq 9$ we have that $BC + B + C \leq 99$ and $C \leq 9$ with equality if and only if $B = C = 9$. Therefore the equation above can only be satisfied if $B = C = 9$. In this case any A will do.