

# Cal Poly Department of Mathematics

## Puzzle of the Week

Feb 12-18, 2010

Here's another puzzle involving three-digit numbers: For every three-digit number  $n = ABC$  define

$$S(n) = A + B + C + (A \times B) + (A \times C) + (B \times C) + (A \times B \times C)$$

Find all (three-digit) numbers  $n$  with  $S(n) = n$ .

*Solutions should be submitted to Morgan Sherman:*

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Office: bldg 25 room 310*

*before next Friday. Those with correct and complete solutions will have their names listed in next week's email announcement. Anybody is welcome to make a submission.*

### Solution:

The number  $n$  must be of the form (in decimal notation)  $x99$  for some digit  $x$ .

I borrowed this puzzle from Brian Miceli – a friend of our own Jeff Liese. Writing  $n$  in decimal notation as “ $ABC$ ” for digits  $A, B, C$  we see that the equation  $n = S(n)$  can be written as

$$100A + 10B + C = A + B + C + AB + AC + CB + ABC$$

which simplifies to

$$99A + 9B = AB + AC + BC + ABC$$

or, more suggestively,

$$99A + 9B = (BC + B + C)A + (C)B.$$

Since  $B, C \leq 9$  we have that  $BC + B + C \leq 99$  and  $C \leq 9$  with equality if and only if  $B = C = 9$ . Therefore the equation above can only be satisfied if  $B = C = 9$ . In this case any  $A$  will do.