

Cal Poly Department of Mathematics

Puzzle of the Week

January 22 - 28, 2010

A pasta dish contains n strands of spaghetti. A diner randomly picks two ends of strands (possibly of the same strand) and glues them together (with edible paste). She then repeats doing this until there are no more free ends to glue together. What is the expected number of loops she will end up with?

Solutions should be submitted to Morgan Sherman:

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before next Friday. Those with correct and complete solutions will have their names listed in next week's email announcement. Anybody is welcome to make a submission.

Solution: Lawrence Sze tells me this question was given to former Cal Poly student Jessica Faruque on her PhD qualifying exam at Stanford!

The expected number of loops is $\sum_{k=1}^n \frac{1}{2k-1}$. Let $E(n)$ denote the expected number and argue by induction:

When $n = 1$ this is clear. Otherwise suppose there are $n + 1$ strands of spaghetti. After grabbing one end there is a $1/(2n + 1)$ chance of grabbing the other end, while a $2n/(2n + 1)$ chance of gluing to another strand. In the former case we end up with $E(n) + 1$ loops. In the latter we will have a total of n strands (one with twice the length of the others) and so we'll end up with $E(n)$ loops. Thus

$$E(n + 1) = \frac{1}{2n + 1}(E(n) + 1) + \frac{2n}{2n + 1}E(n) = \sum_{k=1}^{n+1} \frac{1}{2k - 1}$$

by induction.