

Cal Poly Department of Mathematics

Puzzle of the Week

November 13 - 19, 2009

Basketball star Wane Dwyade makes one of his first two free-throw attempts of his career. Thereafter the probability that he hits the next is equal to the proportion of free throws he has made so far. What is the probability that he hits exactly 50 of his first 100 attempts? How about exactly 1005 of his first 2009?

Solutions should be submitted to Morgan Sherman:

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before next Friday. Those with correct and complete solutions will have their names listed in next week's email announcement. Anybody is welcome to make a submission.

Solution: This problem is based on a 2002 Putnam exam problem (in the original version the basketball player was "Shanille O'Keal").

Wane's chances are $1/99$ and $1/2008$, respectively. To see this we will prove that the chance Wane hits k of his first n free throws ($0 < k < n$) is $\frac{1}{n-1}$. We can do this by induction, with the base case $n = 2$ already established.

To make k of his first n there are two possibilities:

1. Wane made k of his first $n - 1$ and then missed his n th attempt. By induction this occurs with probability $\frac{1}{n-2} \times \frac{n-1-k}{n-1}$.
2. Wane made $k - 1$ of his first $n - 1$ and then made his n th attempt. Again by induction this happens with probability $\frac{1}{n-2} \times \frac{k}{n-1}$.

That means the probability of making k out his first n is

$$p = \frac{1}{n-2} \times \frac{n-1-k}{n-1} + \frac{1}{n-2} \times \frac{k-1}{n-1} = \frac{(n-1-k) + (k-1)}{(n-2)(n-1)} = \frac{1}{n-1}$$

Therefore the result follows by induction.