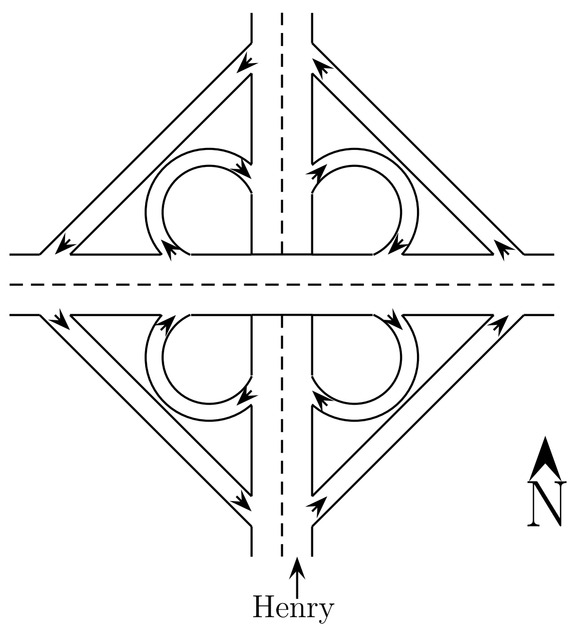
Cal Poly Department of Mathematics

Puzzle of the Week

November 6 - 12, 2009

Henry is driving north on Wall Street and wishes to take the high road and make a left onto Main Street. The intersection is engineered as a clover-leaf interchange – see the figure below. The problem is Henry is test-driving his newest invention: an auto-piloted car. Typical of Henry's inventions it is not working properly and instead of obeying his commands the car is randomly choosing whether to turn right onto any off-ramps. If at every opportunity Henry's car will exit right with probability $\frac{1}{3}$ and continue straight with probability $\frac{2}{3}$ what are Henry's chances of ending up heading west?



Solutions should be submitted to Morgan Sherman:

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before next Friday. Those with correct and complete solutions will have their names listed in next week's email announcement. Anybody is welcome to make a submission.

Solution: This problem was suggested to me by Tom O'Neil.

Notice that one way for Henry to ultimately head west is to bypass the first off-ramp, take the second, and bypass the third. That happens $(\frac{2}{3})(\frac{1}{3})(\frac{2}{3}) = \frac{4}{27} \approx 14.81\%$ of the time. However that is not the only way Henry can get there. After getting on Main St (an odd name, given the interchange is more like a highway interchange) going west Henry could exit south as long as he also takes the next three consecutive exits, which will put him back on Main Street, westbound. He then could skip the next exit, or he could take it as long as he once again circuits around each of the clover-leaf exits. Etc. In total Henry will head left with probability

$$\begin{aligned}
p &= \frac{2}{3}\frac{1}{3}\frac{2}{3} + \frac{2}{3}\frac{1}{3}\left(\frac{1}{3}\right)^4\frac{2}{3} + \frac{2}{3}\frac{1}{3}\left(\frac{1}{3}\right)^8\frac{2}{3} + \dots \\
&= \frac{4}{9}\sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^{4n} \\
&= \frac{4}{9}\sum_{n=0}^{\infty}\left(\frac{1}{81}\right)^{n} \\
&= \frac{4}{9}\frac{1}{1-\frac{1}{81}} \\
&= \frac{4}{9}\frac{81}{81-1} \\
&= \frac{3}{20}
\end{aligned}$$

or 15%.