Cal Poly Department of Mathematics

Puzzle of the Week

October 9 - 15, 2009

A dart is thrown randomly and hits a square dart board. What is the probability that it lands closer to the center than to any edge?

Note/Hint: The answer can be simplified into the form $\frac{a\sqrt{b}+c}{d}$ where a, b, c, d are integers.

Solutions should be submitted to Morgan Sherman:

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before next Friday. Those with correct and complete solutions will have their names listed in next week's email announcement. Anybody is welcome to make a submission.

<u>*Solution:*</u> The probability is $\dfrac{4\sqrt{2}-5}{3}$. To see this choose units and a cartesian coordinate system so that the square has vertices at the four points whose x and y coordinates are ± 1. By symmetry we may assume the dart lands in that triangle with vertices (0,0), (-1,1), (1,1). Here the closest edge for any point is the top edge. Therefore the distance from (x, y) to the edge will be $1 - y$ while its distance to the center is given by the familiar formula $\sqrt{x^2 + y^2}$. Thus the curve separating points closer to the center than those closer to the edge satisfies $(1 - y)^2 = x^2 + y^2$ which we recognize as the parabola $y = \frac{1}{2}(1 - x^2)$. This meets the boundary lines $y = \pm x$ at $(\pm(\sqrt{2} - 1), \sqrt{2} - 1)$. So the region of points closer to the center has area

$$\int_{-(\sqrt{2}-1)}^{\sqrt{2}-1} \frac{1}{2}(1 - x^2)\, dx - (\sqrt{2} - 1)^2 = \frac{4\sqrt{2} - 5}{3}.$$

Since the area of the whole triangle is 1 we take the ratio of the two to get the answer above.

Note: This problem appears in the 1989 Putnam exam. Lawrence Sze informs me this was one of eight problems he was asked to solve during a job interview (he was offered the job). And who says the Puzzle of the Week isn't practical?