Cal Poly Department of Mathematics

Puzzle of the Week

October 2 - 8, 2009

Show that for $n \geq 6$ any square can be partitioned into n sub-squares (not necessarily all of the same size).

Solutions should be submitted to Morgan Sherman:

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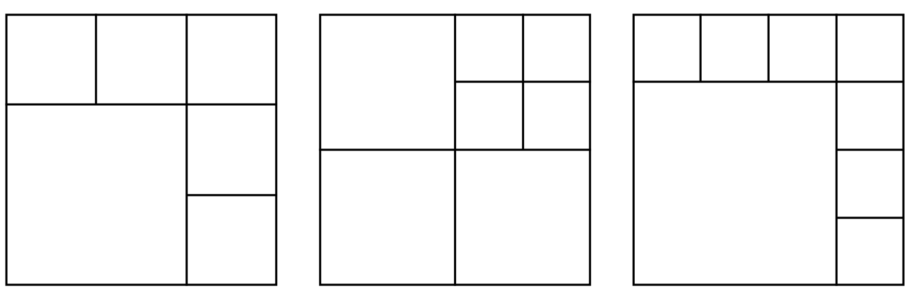
before next Friday. Those with correct and complete solutions will have their names listed in next week's email announcement. Anybody is welcome to make a submission.

Solution: I found this puzzle while browsing Ravi Vakil's (old) Putnam Seminar web page. Note that one square can be made into four sub-squares by dividing it with a vertical and a horizontal line. This means that if we can divide a square into n squares we can also divide it into $n+3$ by performing such a division on any one of its sub-squares. By induction if we can divide into n we can divide into $n+3k$ squares for every $k \geq 1$. It remains only to show the problem can be solved for $n = 6, 7, 8$:



$n = 6$ $n = 7$ $n = 8$

Note: If we define S to be the set of positve integers n such that a square can be divided into n sub-squares then we have shown that every $n \geq 6$ is in S. It is not too hard to show that the only remaining elements of S are 1 and 4, thus completely determining S. Bob Wolf (whose name I misspelled in the weekly email – apologies Bob!) reports that Michael Beeson has recently written a paper which solves a similar problem for triangles: determining the set T of positive integers n such that *some* triangle can be partitioned into n *congruent* triangles.