Write the number 100 as a sum of positive integers by using each of the digits zero through nine exactly once... or prove that it cannot be done.

Note: an example of such a sum (not totaling 100) would be

\[ 32 + 9 + 76 + 10 + 4 + 8 + 5 = 144 \]

Solutions should be submitted to Morgan Sherman:

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before next Friday. Those with correct and complete solutions will have their names listed in next week’s email announcement. Anybody is welcome to make a submission.

\[ x + y = 45 \]
\[ x + 10y = 100 \]

Solving for \( y \) gives \( y = \frac{55}{9} \) which is not an integer, a contradiction.

Note: another approach uses the power of modular arithmetic to note that any such sum is congruent to \( 0 + 1 + 2 + 3 + \ldots + 9 = 45 \) modulo 9. Since \( 45 \equiv 0 \) and \( 100 \equiv 1 \) modulo 9 the sum cannot be 100.

Second Note: Jeff Liese, in an effort to find a completely different approach to the problem, gives the following short argument: The coefficient of \( x^{100} \) in \( (x + x^{10})(x^2 + x^{20})(x^3 + x^{30}) \cdot \ldots \cdot (x^9 + x^{90}) \) is zero and therefore the sum can never be 100. I’ll leave it to the reader to (1) verify the calculation and (2) understand why it works! (Very clever Dr. Liese....)