Cal Poly Department of Mathematics

Puzzle of the Week

September 25 - October 1, 2009

Write the number 100 as a sum of positive integers by using each of the digits zero through nine *exactly* once... or prove that it cannot be done.

Note: an example of such a sum (not totaling 100) would be

$$32 + 9 + 76 + 10 + 4 + 8 + 5 = 144$$

Solutions should be submitted to Morgan Sherman:

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before next Friday. Those with correct and complete solutions will have their names listed in next week's email announcement. Anybody is welcome to make a submission.

<u>*Solution:*</u> I found this problem in Polya's classic book: "How to solve it".

Such a sum can never equal 100.

Here is one of many possible solutions: Let x be the sum of those digits appearing in the ones place and y the sum of those appearing in the tens place. Assuming we sum to 100 then we have two equations

$$x + y = 45$$
$$x + 10y = 100$$

Solving for y gives $y = 55/9$ which is not an integer, a contradiction.

Note: another approach uses the power of modular arithmetic to note that any such sum is congruent to $0 + 1 + 2 + 3 + \ldots + 9 = 45$ modulo 9. Since $45 \equiv 0$ and $100 \equiv 1$ modulo 9 the sum cannot be 100.

Second Note: Jeff Liese, in an effort to find a completely different approach to the problem, gives the following short argument: The coefficient of x^{100} in $(x + x^{10})(x^2 + x^{20})(x^3 + x^{30}) \cdot \ldots \cdot (x^9 + x^{90})$ is zero and therefore the sum can never be 100. I'll leave it to the reader to (1) verify the calculation and (2) understand why it works! (Very clever Dr. Liese....)