Cal Poly Department of Mathematics

Puzzle of the Week
May 22 - 28, 2009

In the Clevland Cavaliers' first game of the 2008-09 season we witnessed LeBron James make 4 out of his 8 free throw attempts, thus ending the game shooting 50% from the stripe. After the last game of the season LeBron finished with a season average of 78% of free thows made.

Question: In going from 50 to 78 percent must LeBron have averaged exactly 75% at some point in time after game 1 and by the end of the season?

Solutions should be submitted to Morgan Sherman:

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before next Friday. Those with correct and complete solutions will have their names listed in next week's email announcement. Anybody is welcome to make a submission.

Solution: This problem comes from the 2004 Putnam exam.

The answer is yes, LeBron did have to actually hit exactly 75% of his free throws at some point. To see this suppose that he managed to go from under 75% to over 75% without hitting exactly 75%. Consider the free throw which sent him over the threshold. If a denotes the number of made free throws just before this point, and b denotes total number of attempts at that time, then those numbers satisfy

$$\frac{a}{b} < \frac{3}{4} < \frac{a+1}{b+1}$$

But then $4a < 3b < 4a + 1$ which is impossible since a and b are integers.

Note that if we replace $3/4$ with $(k-1)/k$ then the same reasoning will still hold. So LeBron would also have made exactly 2/3 of his shots at some point. And any basketball player who goes from 0% to 91% (say) will have made exactly 1/2, 2/3, 3/4, 4/5, 5/6, 6/7, 7/8, 8/9, 9/10 and 10/11 of his or her shots at some point.

Thanks for following the Cal Poly "Puzzle of the Week" this year. It will return in the Fall, so stay tuned..