

# Cal Poly Department of Mathematics

## Puzzle of the Week

May 15 - 21, 2009

Suppose Team  $A$  and Team  $B$  are to play a “best of  $n$ ” games series, where  $n$  is an odd positive integer, and Team  $A$  has a probability  $p > 1/2$  of beating Team  $B$  in any given game.

Since Team  $A$  has the edge in any given game, the greater  $n$  is the greater the over-all chances for Team  $A$ . For example if  $p = .75$  then Team  $A$  has a 75% chance of winning a best of 1 series, but an 84% chance of winning a best of 3 series.

This week’s puzzle asks you to find a formula for Team  $A$ ’s chances for a best of  $n$  series, for an arbitrary  $p > 1/2$  and odd positive integer  $n$ . As a follow up calculate the smallest (odd)  $n$  so that a team with a 51% edge per game has a greater than 99% edge of winning a best of  $n$  series.

*Solutions should be submitted to Morgan Sherman:*

*Dept. of Mathematics, Cal Poly  
Email: sherman1 -AT- calpoly.edu  
Office: bldg 25 room 310*

*before next Friday. Those with correct and complete solutions will have their names listed in next week’s email announcement. Anybody is welcome to make a submission.*

*Solution:* Let  $n = 2m - 1$ . Then to win a best of  $n$  series team  $A$  must win  $k$  games, for some  $k = m, m + 1, \dots, 2m - 1$ . For any particular choice of  $k$  out of the  $2m - 1$  games the chance that  $A$  wins all of them, while  $B$  wins all the others, is  $p^k(1 - p)^{2m-k-1}$ . Since there are  $\binom{2m-1}{k}$  choices for such games and  $A$  wins the series for any  $k \geq m$ , the total probability that  $A$  is the over-all winner is

$$\sum_{k=m}^{2m-1} \binom{2m-1}{k} p^k (1-p)^{2m-k-1}.$$

Now if  $p = 0.51$  then we can plug in the above formula for various values of  $m$ . Trial and error finds that the smallest value for  $m$  which gives team  $A$  a 99% chance of winning the series is 6764. This corresponds to  $n = 13527$ , so the shortest “best of  $n$ ” series which gives team  $A$  a 99% edge overall is a “best of 13527”.

Kent Morrison points out that the above formula can be expressed in terms of the “regularized beta function”, which Mathematica knows. Using this together with a Find Root command one can quickly locate the smallest  $m$  value (6764).