

# Cal Poly Department of Mathematics

## Puzzle of the Week

May 8 - 14, 2009

On a very long, circular, isolated driving route there are a number of small gas stations, each with a limited supply of gasoline left. In total they contain exactly the amount of gasoline needed to get my car around one full circuit. Prove that, starting with an empty tank, there is some station I can start at which will allow me to complete one trip around the route.

Of course I will be collecting gasoline whenever I pass a station. Assume my car's tank is large enough to hold as much as necessary, and that no other cars will be taking any of the available gasoline.

*Solutions should be submitted to Morgan Sherman:*

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Office: bldg 25 room 310*

*before next Friday. Those with correct and complete solutions will have their names listed in next week's email announcement. Anybody is welcome to make a submission.*

*Solution:* This problem appears in Laszlo Lovasz's classic text "Combinatorial Problems and Exercises" as problem 23 from section 3. There are a few ways to go about solving it, and among submitted solutions the most popular seems to be using induction on the number of gas stations. Here is another way to do it: Imagine you have plenty of gas in the tank and drive the circuit picking up gas at all the stations. When you return to the point of origin you will have the same amount of gas as when you left, by hypothesis. However at some point along the way the amount of gas in the tank would have been at an absolute minimum (which must be at a gas station). Starting with an empty tank at this point we could make it all the way around the route.