Cal Poly Department of Mathematics

Puzzle of the Week

May 1 - 7, 2009

Seven sprinters line up for the 100 meter dash. Assuming ties are permitted in how many distinct ways can all seven runners cross the finish line? For example: Runners A through G might finish

B first, G second, ACF three-way tie for third, and DE tied for last.

Solutions should be submitted to Morgan Sherman:

Dept. of Mathematics, Cal Poly
Email: sherman1 -AT- calpoly.edu
Office: bldg 25 room 310

before next Friday. Those with correct and complete solutions will have their names listed in next week's email announcement. Anybody is welcome to make a submission.

<u>*Solution:*</u> If one knows about Stirling numbers the problem isn't too difficult. But even without them we can work out the case for seven runners: Let $T(n)$ denote the number of way n runners can finish. Imagine the runners finishing in "blocks", each block consisting of runners finishing in a tie. If the first block has k runners in it then this can happen in $\binom{n}{k}T(n-k)$ ways, where $\binom{n}{k}$ is the number of ways of choosing k runners from a group of n. Then we get the recursive formula

$$T(n) = \sum_{k=1}^{n} \binom{n}{k} T(n-k)$$

where we must set $T(0) = 1$. From this we can calculate:

n	1	2	3	4	5	6	7
$T(n)$	1	3	13	75	541	4683	47293

So there are 47,293 ways seven runners can finish the race.