Seven sprinters line up for the 100 meter dash. Assuming ties are permitted in how many distinct ways can all seven runners cross the finish line? For example: Runners A through G might finish

    B first, G second, ACF three-way tie for third, and DE tied for last.

Solutions should be submitted to Morgan Sherman:

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before next Friday. Those with correct and complete solutions will have their names listed in next week’s email announcement. Anybody is welcome to make a submission.

Solution: If one knows about Stirling numbers the problem isn’t too difficult. But even without them we can work out the case for seven runners: Let $T(n)$ denote the number of way $n$ runners can finish. Imagine the runners finishing in “blocks”, each block consisting of runners finishing in a tie. If the first block has $k$ runners in it then this can happen in $\binom{n}{k}T(n-k)$ ways, where $\binom{n}{k}$ is the number of ways of choosing $k$ runners from a group of $n$. Then we get the recursive formula

$$T(n) = \sum_{k=1}^{n} \binom{n}{k}T(n-k)$$

where we must set $T(0) = 1$. From this we can calculate:

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T(n)$</td>
<td>1</td>
<td>3</td>
<td>13</td>
<td>75</td>
<td>541</td>
<td>4683</td>
<td>47293</td>
</tr>
</tbody>
</table>

So there are 47,293 ways seven runners can finish the race.