

Cal Poly Department of Mathematics

Puzzle of the Week

Jan 23 - 29, 2009

An infinite chess board has squares of side length 1. A coin with radius $r < \frac{1}{2}$ is flipped onto the board. What is the probability (in terms of r) the coin intersects an edge?

Solutions should be submitted to Morgan Sherman:

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before next Friday. Those with correct and complete solutions will have their names listed in next week's email announcement. Anybody is welcome to make a submission.

Solution: The coin will land on an edge with probability $4r(1 - r)$.

Consider the square S in which the center C of the coin lands, and set up coordinates so that it has vertices at $(0, 0)$, $(1, 0)$, $(0, 1)$, $(1, 1)$ and suppose $C = (x, y)$. Then the coin will lie completely inside the square precisely if both $r < x < 1 - r$ and $r < y < 1 - r$. This describes a square with area $(1 - 2r)^2$. Thus the probability the coin will not intersect an edge is ratio of this area to that of the S . Therefore the probability that the coin *does* intersect an edge is $1 - (1 - 2r)^2/1$, which reduces to $4r(1 - r)$. [Note: Kent Morrison points out that the coin missing a horizontal line and the coin missing a vertical line are independent events, leading to another way to approach the problem.]

Source: "Mathematics and Chess" by Miodrag Petkovic