Physics 211 Winter 2009

Relativity Problems

Assignment I - due Wednesday January 14

TZD Chapter 1: through section 10, including the examples; or Mermin: Ch 1-7
Problems: 20, 23, 26a&b, 30
From the problems below: 3, 4, 5, 10 (1 was done in class) start thinking about 2

1. This is the problem that we did in class to show that 'moving rods shrink': (see Mermin chapter 6) We found that a light clock (proper length L_o) moving perpendicular to its length takes a time $2\gamma L_o/c$ to tick once. If the clock is turned on its side so that it moves parallel to its length, it must take the same amount of time to tick. Why? Use this fact to show that the length of this moving clock must be $L = L_o/\gamma$, where L is the measured length of the moving clock. Hint: Begin by finding the time needed for a light flash emitted by one end of the clock to reach the other end of the (moving) clock.

2. A flat, horizontal pan of dough speeds under a circular cookie cutter at a velocity near c. A baker, holding the cutter perfectly horizontally, stamps the dough with lightning speed. (Assume he raises the cutter again so quickly that nothing gets squashed or stuck in the cutter.) The resulting cookie will not be circular, but elliptical. (By the shape of a cookie, one means, of course, its shape in its proper frame.) Is it longer in the direction of its motion or the perpendicular direction? Justify this in the frame of the baker and in the frame of the dough.

3. Alpha Centauri is a star about 4 light-years away. For a rocket to make the trip in one year (as reckoned by its occupants), how fast must it travel? To the occupants of the rocket Alpha-Centauri would appear to be approaching at the same speed. How far away does it appear, to them, to be as they start the trip?
   How fast must the rocket travel in order to make the trip in one day?

4. A rocket ship of length Lo (in the rocket's frame) leaves the earth at a speed v. A light signal is sent after it which arrives at the rocket's tail at t' = 0 according to rocket clocks and t = 0 according to earth clocks.
   a. When does the signal reach the head of the rocket according to rocket clocks?
   b. When does the signal reach the head of the rocket according to earth clocks?

   The light signal is reflected from the head of the rocket back to the tail.
   c. How long does it take to reach the tail of the ship according to rocket clocks? What do the rocket clocks read now?
   d. How long does the return trip take according to earth clocks? What does the earth clock at the tail now read?

5. If a train of length L is stationary, then light from a bulb flashed at the midpoint of the train will reach the two ends at the same time. If the train moves past a station platform at speed v, then (in the frame of the station) the light from a single flash of this bulb will no longer reach the two ends of the train at the

same time, since the rear end is moving toward the light while the front end is moving away. Evidently if we want to place the bulb so the light reaches the ends of the train simultaneously in the frame of the station, we must move the bulb closer to the front of the train; i.e., we must attach it at a point a fraction x of the total length of the train behind the front, where x is less than 1/2. What is x? (Nothing is required to do this but the fact that the speed of light is c in the frame of the station.) Your answer should reduce to the obvious answer x = 1/2 when v = 0 and have the very reasonable property of requiring the bulb to get closer to the front of the train, as v gets closer to c. Find an algebraic expression for x (in terms of v).

Does x equal 1/2 when v = 0?

6. We know that u_x and $u_{x'}$ are related by

$$u_x = \frac{u_{x'} + v}{1 + u_{x'} v/c^2}$$

If a beam of light is moving in the vertical (y') direction in the primed frame (which moves at speed relative v to the unprimed frame), what are $u_{x'}$ and $u_{y'}$ for this beam? Find u_x for the beam? Use this answer and the constancy of the speed of light to find u_y? How is all of this related to our original analysis of the light clock?

7. The alleged traffic violator, in good conscience, tells the judge that the top light on the signal was green. Is the defendant guilty? (Find the speed)
Note: $\lambda_{red} \approx 700$ nm and $\lambda_{green} \approx 500$ nm

8. The sun uses the reaction:

$$4p \text{ -----> } {}^4\text{He} + \text{energy} + \text{particles}$$

to generate 4×10^{26} watts. For each ^{4}He nucleus produced, the 'liberated' energy is 26.7 MeV. How many ^{4}He must the sun produce each second? How much mass does the sun lose each year? If the sun ($M = 2 \times 10^{30}$ kg) began as 75% protons, how long can the sun last? In fact it will only last about one tenth this long.

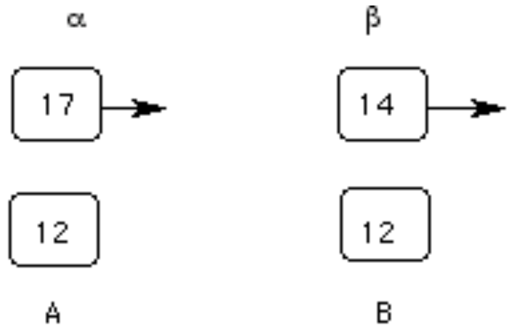
9. A pion will decay into two photons:

$$\pi \text{ ----> } \gamma + \gamma$$

The rest energy of a pion is 135 Mev.
Consider this decay as seen in the rest frame of the pion. Conservation of momentum tells us that the photons are emitted in opposite directions (we will take those directions to be along the x' axis).
a. What does this tell us about how the energies of the two photons compare?
b. What is the energy of each photon?

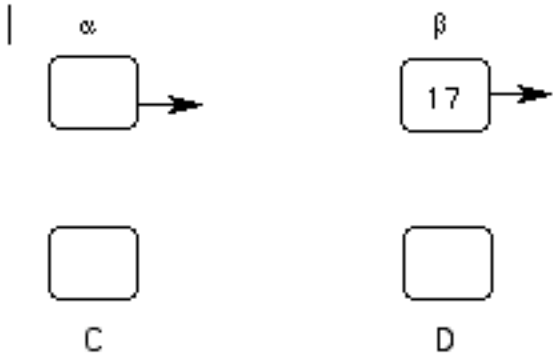
As viewed in a frame (the lab frame) in which the pion is moving along the x axis at speed u = .6c :
c. What must the total energy of the two photons equal?
d. Use the fact that the energy of a photon is Doppler shifted (in the same way as the frequency of light is) to find the energy of the photon emitted in the forward direction.

10. In this picture clocks A and B are at rest and are 4 light seconds apart. The clocks all read seconds. The moving clocks (α and β) go by at speed v.

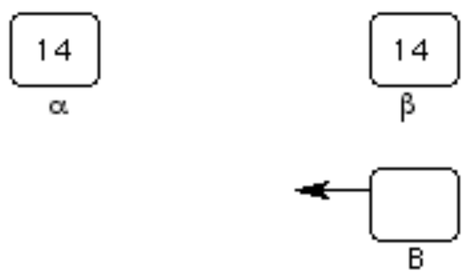


a. Find v. What is the proper separation of α and β ?

b. A later picture is shown below. Clocks C and D are also at rest with respect to A and B. Fill in the blank clocks.
What is the proper separation of clocks A and C?



c. As seen by observers at rest with respect to α and β



What does clock B read?
Where is clock A and what does it read?
Fill in the figure.