1. A park down the road has a 22.0 m long slide (angled 35 degrees above the horizon) with a spring (k=4500 N/m) at the bottom. A 45.0 kg child rebounds from the spring and slides back up the slide. Assume the region on the slide where the spring compresses is frictionless but the rest of the slide has coefficients of friction with the child's clothing given by \( \mu_s = 0.2 \) and \( \mu_k = 0.5 \).

   a) **Qualitative**: Comment with a brief explanation) on the sign (positive, negative, or zero) for the work done by friction, normal force and gravity on the way down the slide and on the way back up the slide.

   b) **Qualitative**: Can the maximum height to which the child rebounds be determined without considering what happens with the spring? Explain.

   c) If the child starts from the top of the slide, what is the speed of the child on the way down when the spring is compressed 0.50 m? After the spring compresses all the way and the child is rebounding back up the slide, will the child have the same speed as the way down when the spring is compressed 0.50 m? Explain.

\[
\begin{align*}
\text{Frictionless} & \\
\text{Gravity} & \\
\text{Normal} & \\
\text{Spring} & \\
\end{align*}
\]

\[
\begin{align*}
\theta = 35^\circ & \\
\text{Spring} & \\
\text{Child} & \\
\end{align*}
\]

\[
\begin{align*}
W_{\text{friction}} &= \frac{f_k \cdot \Delta \vec{r}}{f_k} \cdot \Delta \vec{r} = f_k \Delta \vec{r} \cos 180^\circ = (-) \quad \text{(friction opposes the displacement both in the way up and down)} \\
W_{\text{gravity}} &= m g \cdot \Delta \vec{r} = m g \Delta \vec{r} \cos 35^\circ = (+) \quad \text{(\( F_{\text{ grav }} \) is in direction of motion on the way down)} \\
W_{\text{normal}} &= N \cdot \Delta \vec{r} = N \Delta \vec{r} \cos 90^\circ = 0 \quad \text{(\( N \) and \( \Delta \vec{r} \) are \( \perp \) both on way up and down)}
\end{align*}
\]
b) Yes we can determine the maximum height! Since the spring is assumed to be conservative and \( W_{fr.net} = 0 \) as the spring compresses and uncompresses, we have:

\[ W_{fr.net} = \Delta U_s - \Delta U_y = \Delta K. \]

\[ \Delta K = 0, \text{ since the initial and final speeds are zero.} \]

\[ \Delta U_y = 0, \text{ since the initial and final positions are the same.} \]

\[ W_{fr.net} = \Delta U_y \] which can be used to determine the maximum height.

\[ \Delta U_y = 0.5 \times m \times v_{fr.max}^2 \]

\[ \Delta U_y = 0.5 \times m \times [22.5 \times \text{m/s}]^2 \]

\[ \Delta U_y = 6033.75 \text{ J} \]

\[ \frac{1}{2} \times m \times v_{fr.max}^2 = 0.5 \times m \times 6033.75 \]

\[ v_{fr.max}^2 = 6033.75 \]

\[ v_{fr.max} = 77.62 \text{ m/s} \]

\[ h = \frac{1}{2} \times 9.8 \times 0.5^2 = 1.225 \text{ m} \]

\[ V_f = \sqrt{\frac{2 \times g \times 0.5 \times (\sin 35^\circ - 0.5 \times \cos 35^\circ)}{0.5 \times \cos 35^\circ} - 45000} \]

\[ V_f = 12.5 \text{ m/s} \]

This speed is the same on the way up when the spring is compressed 0.5 m because only conservative forces act.
2. Consider the following force versus time graph:

a) A 500 g particle is moving to the left along the x-axis with a speed of 2 m/s at \( t = 0 \) s. The particle then experiences the force shown graphically in the figure. What is the particle's speed and direction of motion at \( t = 3 \) s?

\[
\Delta P = \int_{t_i}^{t_f} F_x \, dt = \text{Area}
\]

\[\text{m}V_f - \text{m}V_i = \text{Area}\]

\[V_f = V_i + \frac{\text{Area}}{m}\]

\[V_f = -2.0 + \frac{4.5}{0.5} = 7.0 \text{ m/s}
\]

b) A 500 g particle is moving to the right along the x-axis with a speed of 2 m/s at \( x = 0 \) m. The particle then experiences the force shown graphically in the figure. What is the particle's speed and direction of motion at \( x = 3 \) m?

\[
\Delta k = W_{net} = \int \underline{F} \cdot ds
\]

\[\frac{1}{2} \text{ m}V_f^2 - \frac{1}{2} \text{ m}V_i^2 = \text{Area}\]

\[V_f^2 = V_i^2 + 2 \cdot \text{Area} \cdot \frac{1}{m}\]

\[V_f = \sqrt{V_i^2 + 2 \cdot \frac{9.0 \text{ N} \cdot \text{s}}{0.5}} = 4.69 \text{ m/s}
\]
3. A 1.0 kg mass is attached to a 1.5 m string (which is attached to the ceiling). The ball is released from rest so that the initial angle of the string is $\theta_1 = 55^\circ$. At the bottom of the trajectory the ball collides with a 3.0 kg block. After the collision the 1.0 kg mass rebounds so that the string makes an angle $\theta_2 = 18^\circ$.

![Diagram showing the initial and final states of the system](image)

a) What is the speed of the 3.0 kg block just after the collision?
b) Determine the type of collision that takes place between the ball and block. Be sure to justify your answer numerically.

**For $V_{1f}$:**

Conservation of Energy

\[ K_i + U_i = K_f + U_f \]

\[ m g y_i = \frac{1}{2} m v_f^2 \]

\[ v_f = \sqrt{2 g y_i} = \sqrt{2 \cdot 9.8 \cdot (1 - \cos 55^\circ)} = \sqrt{2 \cdot 9.8 \cdot 1.5 (1 - \cos 55^\circ)} \]

\[ v_f = V_{1f} = 3.54 \text{ m/s} \]

**For $V_{1f}$:**

Same thing gives

\[ V_{1f} = \sqrt{2 \cdot 9.8 \cdot (1 - \cos 18^\circ)} = \sqrt{2 \cdot 9.8 \cdot 1.5 (1 - \cos 18^\circ)} \]

\[ V_{1f} = 1.20 \text{ m/s} \]

**For $V_{2f}$:**

Conservation of Momentum

\[ m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \]

\[ V_{2f} = \frac{m_1 v_{1i} - m_1 v_{1f}}{m_2} = \frac{(1)(3.54) - (1)(-1.2)}{3.0} \]

\[ V_{2f} = 1.58 \text{ m/s} \]
b) \[ K_i = \frac{1}{2} m_1 v_{i1}^2 = \frac{1}{2}(1.0)(3.94)^2 = 6.27 \text{ J} \]

\[ K_f = \frac{1}{2} m_1 v_{f1}^2 + \frac{1}{2} m_2 v_{f2}^2 \]

\[ = \frac{1}{2}(1)(2)^2 + \frac{1}{2}(3)(1.58)^2 \]

\[ K_f = 4.46 \text{ J} \]

Since \( K_i \neq K_f \Rightarrow \text{Not perfectly elastic} \)

or it is an inelastic collision.