Sharpe Ratio over investment Horizon

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ABSTRACT

Both building blocks of the Sharpe ratio — the expected return and the expected volatility — depend on the investment horizon. This raises a natural question: how does the Sharpe ratio vary with horizon? To address this question, we derive an explicit expression of the Sharpe ratio as a function of the investment horizon. Even if returns are independent and normally distributed, we show that the Sharpe ratio is humped shaped — it first increases and then decreases as the horizon lengthens. We empirically corroborate our theory by analyzing the Sharpe ratio of various funds.

Our analysis has an important implication for investors. Specifically, it is a common practice for investors to rank funds based on their Sharpe ratio. We show that the order of funds ranked by the Sharpe ratio may change with horizon. Namely, it may appear that one fund has a higher Sharpe ratio than other when the horizon is three years, but that the opposite holds true when the horizon increases to five years. Therefore, investors need to consider the “Horizon Effect” while considering the Sharpe ratio based rankings.

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I. Introduction

Sharpe ratio is probably the most popular index of measuring a portfolio’s reward to risk performance. Its popularity lies in its inherent simplicity: it is the ratio of the expected excess return of an investment to its volatility. Because both the expected return and volatility depend on the investment horizon, the Sharpe ratio depends on the horizon also. That being said, the relationship between the Sharpe ratio and horizon is not well-understood. In this article, we analyze the relationship between the Sharpe ratio and investment horizon.

Our analysis has an important implication for investors. While elegantly summarizing the use of the Sharpe ratio, Kidd (2011) offers a simple rule of thumb: *Higher is better*. That is, in the process of comparing two funds, the fund with the higher Sharpe ratio is preferred. We show that this simple rule of thumb may lead to a bad decision. The error is mechanical in nature: it stems from the impact of horizon on the Sharpe ratio. Specifically, the relative ranking of two funds may change with the investment horizon. We label the change in ranking phenomenon as the “Horizon Effect”.

To motivate our analysis, consider the following example. In Figure 1, we show the Morningstar Sharpe ratio over different horizons for two funds: Vanguard Total Stock Market Index Fund (VTSMX) and Vanguard 500 Index (VFINX). VTSMX is the largest mutual fund in the world with $343 billion in assets under management. VFINX, on the other hand, is the largest passive mutual fund mimicking S&P 500 with $173 billion in assets under management. As expected, the returns of these two funds are almost perfectly correlated (correlation coefficient is 0.98). Inspite of the near perfect correlation, Sharpe ratios are different.

The plot raises a few questions. First, why is the 15-year Sharpe-ratio of both funds lower than the 3-year Sharpe ratio? The second question concerns the Horizon Effect: why does the Sharpe ratio ranking change for these two funds? Specifically, the 3-year Sharpe ratio for VFINX is higher than the 3-year Sharpe ratio for VTSMX. However, the 15-year Sharpe ratio for VFINX is lower than the 15-year Sharpe ratio for VTSMX. These two questions point out

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1The assets under management numbers are based on Morningstar estimates as of August 13, 2014. We chose these two funds due to their size — our results are more general.
the undesirable impact of the horizon on the Sharpe ratio.

Lastly, the general shape of the plot directly violates the implication stemming from the Square root $T$ rule commonly used in practice. Under the assumption of serially uncorrelated returns, in an influential article, Sharpe (1998) offers the Square root $T$ rule:

*Letting $S_1$ and $S_T$ denote the Sharpe Ratios for 1 and $T$ periods, respectively, it follows that:*

$$S_T = \sqrt{T} \times S_1.$$

The third question concerns the applicability of the Square root $T$ rule. Specifically, under what scenario is the Square root $T$ rule applicable?

![Figure 1: This figure shows the Sharpe Ratio for two funds as calculated by Morningstar as of August 12, 2014. VTSMX is the symbol for Vanguard Total Stock Market Index Fund and VFINX is the symbol for Vanguard 500 Index.](chart.png)

This article addresses these questions. For simplicity, we assume that the log-returns are independently and normally distributed. With this assumption, we show that expected return, volatility and in turn the Sharpe ratio are non-linear functions of the investment horizon. Due
to non-linearity, Sharpe ratio increases initially and then decreases as horizon lengthens. Non-linearity also causes a switch in relative rankings (Horizon Effect). To summarize, non-linearity leads to a non-monotonic relationship between the Sharpe ratio and investment horizon.

We show that compounding causes the non-monotonic relationship. As defined, Sharpe ratio depends on the moments of simple returns. Unfortunately, due to compounding, statistics of multi-period simple returns is not so simple. To address the compounding issue, we calculate the Sharpe ratio using log-returns. With this simple change, we show three very desirable properties: (i) Sharpe ratio monotonically increases with horizon; (ii) the relative ranking across horizons does not change and (iii) the Square root $T$ is correct. In this manner, we reconcile the seemingly contradictory academic literature. We explain why on one hand, Levy (1972), Hodges et al. (1997) and Lin and Chou (2003) find that the Sharpe ratio is a non-linear function of the horizon and why on the other hand, Lo (2002) finds that the Sharpe ratio should increase with the Square root $T$ rule when returns are serially independent.\footnote{To be precise, for expositional clarity, Lo (2002) either ignores compounding or assumes continuous compounding (See footnote 7 in the paper). Additionally, both Sharpe (1998) and Lo (2002) recognize the complicated effect of compounding. For instance, Sharpe (1998) recommends using short periods (for example, monthly) to measure risks and returns and then annualizing the data. He believed using multi-period returns complicates the ratio because of compounding or potential serial correlation.}

Lastly, we calculate the Sharpe ratio of various funds using both simple and log-returns and corroborate the impact of horizon. We empirically validate Kidd (2011)'s rule of thumb: when Sharpe ratio is calculated using log-returns, higher is [indeed] better.

II. Model showing the effect of horizon on the Sharpe ratio

This section introduces the model. Traded are an index with time $t$ value denoted by $P(t)$ and a money market account, with time $t$ value denoted by $B(t)$. We start the money market account with a dollar, i.e. $B(0) = 1$. Without loss of generality, we also assume that the index pays no dividends.
We assume that the dynamics of the index are given by:

\[ P(t) = P(0) \exp\{r(t)\} \quad t \geq 0. \quad (1) \]

For each \( t \), the random variable \( r(t) \) is interpreted as the log-return over \( t \) periods. We further assume that the random variable \( r(t) \) is normally distributed:

\[ r(t) \sim \mathcal{N}\left(\left(\mu - \frac{1}{2} \sigma^2\right)t, \sigma^2 t\right) \]

where

- \( \mu \) is interpreted as an instantaneous annualized drift rate,
- \( \sigma \) is interpreted as an instantaneous annualized volatility.

Equation \((1)\) implies that the index price, \( P(t) \), is log-normally distributed.\(^3\)

We also assume that the risk-free rate is constant. That is, the money-market account evolves as:

\[ B(t) = B(0) \exp\{r_f t\}. \quad (2) \]

Over a span of \( T \) periods, the simple return between \( t \) and \( t + T \) is

\[ R(t, t + T) \equiv \frac{P(t + T)}{P(t)} - 1, \quad (3) \]

while the log-return over \( T \) periods is

\[ r(t, t + T) \equiv \ln\{1 + R(t, t + T)\}. \quad (4) \]

Slight algebra shows that the simple returns aggregate as

\[ 1 + R(t, t + T) = \prod_{i=0}^{T-1} \left(1 + R(t + i, t + i + 1)\right) \quad (5) \]

\(^3\)Equation \((1)\) is the same as assuming that the index price dynamics follow a Geometric Brownian motion.
while log-returns aggregate as

\[ r(t, t + T) = \sum_{i=0}^{T-1} r(t + i, t + i + 1). \]  \hspace{1cm} (6)

Equation (6) indicates that a \( T \) period log-return is the sum of \( T \) one-period log-returns. As a result, as long as the log-return is stationary, the exact distribution of the \( T \) period log-return can be derived with relative ease. Alternatively, central limit theorem can be readily applied. On the other hand, working with simple returns is not so simple. Due to the complicated nature of aggregating simple returns in equation (5), conventional statistics are harder to calculate. Therefore, due to the tractable properties, both academics and practitioners implicitly prefer to work with log-returns.

In the appendix, we show that the expected simple and the expected log-return at time \( t \) for a \( T \) period horizon are

\[ \mathbb{E}_t [R(t, t + T)] = \exp\{\mu T\} - 1, \]  \hspace{1cm} (7)

and

\[ \mathbb{E}_t [r(t, t + T)] = (\mu - \frac{1}{2} \sigma^2) T. \]  \hspace{1cm} (8)

Similarly, the variance of simple and the variance log-returns at time \( t \) for a \( T \) period horizon are

\[ \text{Var}_t [R(t, t + T)] = \exp\{2 \mu T\} (\exp\{\sigma^2 T\} - 1), \]  \hspace{1cm} (9)

and

\[ \text{Var}_t [r(t, t + T)] = \sigma^2 T. \]  \hspace{1cm} (10)

With the expression of the expectation and variance of simple and log-returns, we analyze the Sharpe ratio next.
A. Expression for the Sharpe ratio using simple returns

The simple return from investing in the money-market account is

\[ R_f(t, t + T) \equiv \frac{B(t + T)}{B(t)} - 1 = \exp\{r_f T\} - 1. \]

Sharpe ratio over horizon \( T \) using simple returns is

\[
SR^S(T) \equiv \frac{\mathbb{E}_t [R(t, t + T)] - R_f(t, t + T)}{\sqrt{\text{Var}_t [R(t, t + T)]}} = \frac{\exp\{\mu T\} - \exp\{r_f T\}}{\sqrt{\exp\{2\mu T\} (\exp\{\sigma^2 T\} - 1)}}.
\] (11)

(12)

For clarity, we include the superscript \( S \) indicating that the Sharpe ratio is calculated using simple returns. Upon inspection of equation (12), it is clear that both the numerator and denominator of the Sharpe ratio are non-linear functions of the horizon \( T \).

B. Expression for the Sharpe ratio using log-returns

Sharpe ratio over horizon \( T \) using log-returns is

\[
SR^L(T) \equiv \frac{\mathbb{E}_t [r(t, t + T)] - r_f T + \frac{1}{2} \text{Var}_t [r(t, t + T)]}{\sqrt{\text{Var}_t [r(t, t + T)]}} = \frac{(\mu - r_f)}{\sigma} \times \sqrt{T} = SR^L(1) \times \sqrt{T}.
\] (13)

(14)

The variance term in the numerator of equation (13) is a Jensen’s inequality adjustment arising from the expectation of log-returns. Effectively, this term converts the expected excess return from a geometric average to an arithmetic average. For more details, see equation 11 of the Consumption based Asset Pricing handbook chapter in Campbell (2003). Again, for clarity, we include the superscript \( L \) indicating that the Sharpe ratio is calculated using log returns.

Upon inspection of equation (14), the Square root \( T \) rule is clearly evident. In fact, this is the precise expression put forth by Sharpe (1998) and derived rigorously in Lo (2002). Next,
we compare the Sharpe ratio calculated using both simple and log-returns.

**C. Comparison of Sharpe ratios calculated using either simple or log-returns**

By Taylor expanding $SR^S(T)$ as a function of $T$, we see that

\[
SR^S(T) = \sqrt{T} \times \frac{(\mu - r_f)}{\sigma} + T^{\frac{3}{2}} \times \frac{\mu r_f - 0.5(\mu^2 + r_f^2) + 0.25\sigma(r - \mu\sigma)}{\sigma} + O\left[T^{\frac{5}{2}}\right] = SR^L(T) + O\left[T^{\frac{5}{2}}\right]
\]

(15)

Upon inspection of equation (15), the Sharpe ratio calculated using simple returns approximately follows the Square root $T$ rule for short horizons. As horizon $T$ lengthens, higher order terms of $SR^S(T)$ carry more weight. As a result, departures from the Square root $T$ rule become more evident.

Figure 2 shows the Sharpe ratio $SR^S(T)$ calculated using simple returns for two different set of parameters. The solid line plots the ratio assuming the drift rate of the index $\mu$ is 8% and the volatility $\sigma$ is 15%. We label this first set of parameters as Parameter Set 1. These numbers are consistent with S&P 500. The solid line with square markers shows the Sharpe ratio where both drift and volatility are slightly inflated: $\mu$ is 12% and $\sigma$ is 20%. We label this second set of parameters as Parameter Set 2. Additionally, we set the risk free rate to 1%.

Two observations are in order. First, as pointed out by Levy (1972), Hodges et al. (1997) and Lin and Chou (2003), the Sharpe ratio calculated using simple returns $SR^S(T)$ does not monotonically increase with the horizon $T$. In fact, the Sharpe ratio is humped shaped. Second, there is a Sharpe ratio ranking reversal — this is the Horizon Effect. Sharpe ratio using Parameter Set 2 outperforms Sharpe ratio using Parameter Set 1 when the horizon is short. However, the ranking reverses at longer horizons.

Figure 3 shows the Sharpe ratio $SR^L(T)$ calculated using log-returns for the same set of parameters. The results are dramatically different. The Sharpe ratio is a monotonically
Figure 2: This figure compares the Sharpe ratio $SR^S(T)$ calculated by using simple returns as a function of investment horizon $T$.

increasing function of the investment horizon consistent with Lo (2002). Second, there is no ranking reversal. Sharpe ratio using Parameter Set 1 always outperforms Sharpe ratio using Parameter Set 2.

Next, we corroborate our theory by analyzing the Sharpe ratio for various funds over different horizons.

III. Empirical analysis of the Sharpe ratio of various funds over different horizons

To illustrate the effect of horizon, we estimate the Sharpe ratio of several popular portfolios. Specifically, from Ken French’s website, we use monthly returns of portfolios sorted by size and book to market. The sample starts from the year 1927 and ends in year 2013. We sample returns at a monthly frequency as it is a common practice among both industry practitioners and academics.

4In the online appendix, we also present results for portfolios formed on earnings-price ratio and five industry portfolios. We also calculate the Sharpe ratio using data from different sample periods — our results are robust.
Figure 3: This figure compares the Sharpe ratio $SR^L(T)$ calculated by using log-returns as a function of investment horizon $T$.

To improve our estimation, we calculate the Sharpe ratios using bootstrapping with replacement. To calculate the Sharpe ratio for a horizon of $T$ years, we draw with replacement a random sample of $T \times 12$ returns from the pool of all available monthly returns. We use equations (5) and (6) to compute either a $T$ period simple return or a $T$ period log-return. By bootstrapping with replacement, we implicitly assume that the returns are serially uncorrelated. To remedy this issue, we also performed our analysis using block bootstrapping as recommended by Lin and Chou (2003). Results do not change both qualitatively and quantitatively. The Sharpe ratio is calculated for each drawn sample, and this procedure is repeated 100,000 times for each $T$ year horizon. The reported Sharpe ratio for $T$ year horizon is an average of 100,000 calculations. Due to the incredible precision, we do not report standard errors for our estimates.

Table I shows the Sharpe ratio $SR^S(T)$ calculated using simple returns. The number in the parenthesis is the Sharpe ratio calculated using the Square root $T$ rule ($\sqrt{T} \times SR^S(1)$). The Sharpe ratios range from 0.37 (Growth portfolio with horizon of one year) to 0.84 (Large portfolio with horizon of ten years). Upon inspection, the basic non-monotonic shape of Figure I is evident for all portfolios. The 7-year Sharpe ratio is higher than 25-year Sharpe ratio for
all the portfolios. Broadly speaking, the Sharpe ratio increases initially and then decreases as horizon lengthens. Second, the Horizon Effect is also clear: there are several instances of Sharpe ratio ranking reversals. For example, the Growth portfolio looks inferior to the Value portfolio at a horizon of 1-year, but it looks superior at a horizon of 10-years. To summarize, consistent with equation (12), the Sharpe ratios calculated using simple returns are humped shaped and suffer from the Horizon Effect.

<table>
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<tr>
<th>Horizon $T$</th>
<th>Small</th>
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<th>Large</th>
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Table I: This table calculates the Sharpe ratio using simple returns for different horizons. The number in parenthesis shows the hypothetical Sharpe ratio calculated using the Square root $T$ rule.

Table II shows the Sharpe ratio $SR^L(T)$ calculated using log-returns. These results are quite different from those in Table I. The Sharpe ratios range from 0.28 (Small portfolio with horizon of 1 year) to 1.78 (Value portfolio with horizon of 10 years). Upon inspection, the Sharpe ratios increase with horizon for all portfolios. Furthermore, there are no ranking reversals as predicted by equation (14) — the Horizon Effect disappears. As expected from equation (15), the Sharpe ratio calculated using both simple and log-returns are approximately similar for short horizons.

Lastly, note the applicability of the Square root $T$ rule. Using simple returns as is the case in Table I, the Square root $T$ becomes an increasingly bad rule as the horizon lengthens. For example, the 1-year Sharpe ratio, $SR^S(1)$, for the Growth portfolio is 0.37. Applying the Square root $T$ rule for a horizon of 25 years, $\sqrt{25 \times 0.37}$, gives a predicted value of 1.85; while
Table II: This table calculates the Sharpe ratio using log-returns for different horizons. The number in parenthesis shows the hypothetical Sharpe ratio calculated using the Square root $T$ rule.

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The actual Sharpe ratio, $SR^S(25)$, is 0.71. On the other hand, using log-returns as is the case in Table II, the Square root $T$ holds remarkably well. The 1-year Sharpe ratio, $SR^L(1)$, for the Growth portfolio is 0.29. Applying the Square root $T$ rule for a horizon of 25 years, $\sqrt{25} \times 0.29$, gives a predicted value of 1.45; while the actual Sharpe ratio, $SR^L(25)$, is 1.44.

In summary, we demonstrate how the behavior of the Sharpe ratio changes when we use either simple or log-returns. We show that using simple returns as is commonly used in practice, the Sharpe ratio is actually humped shaped. On the other hand, using log-returns, the Sharpe ratio increases monotonically with horizon. Furthermore, using log-returns, Square root $T$ rule holds remarkably well.

**IV. Conclusion**

Although the Sharpe ratio has gained popularity, its dependence on the investment horizon is not well-understood. In this paper, we remedy this issue by analyzing the effect of horizon on the Sharpe ratio. Under the assumption that log-returns are independently and normally distributed, we derive an explicit expression of the Sharpe ratio.
We theoretically and empirically highlight three undesirable features of the Sharpe ratio. First, we show that the Sharpe ratio is not a monotonic function of the horizon. The second feature has an explicit implication for an investor who uses the Sharpe ratio to choose funds. We document the Horizon Effect: Sharpe ratio is subject to a ranking reversal over different horizons. That is, an investor may prefer one fund over short horizon but may prefer another fund over longer horizon. Third, we show that the Square root T rule becomes an increasingly bad rule as horizon lengthens. The undesirable features are due to compounding of simple returns. As noted by both Sharpe (1998) and Lo (2002), compounding taints the use of the Sharpe ratio over longer horizons. In this paper, we formally show how compounding taints the Sharpe ratio over longer horizon.

We conclude with one suggestion. We recommend a slight change: calculate Sharpe ratio using log-returns. This change remedies the three undesirable features. With this change, Kidd (2011)'s rule of thumb regarding the Sharpe ratio is indeed valid: higher is better.
References


