Rent-to-own Housing Contracts under financial constraints

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ABSTRACT

Due to tougher lending standards, Rent-to-Own (RTO) contracts are getting popular. In a RTO contract, a tenant signs a lease with the option to buy a house at a predetermined purchase price. That is, the tenant receives a call option. RTO contracts allow potential financially constrained homebuyers to lock in a price while they use the time to build up savings and creditworthiness. Even ex-chairman Bernanke has endorsed RTO contracts as a tool to stem foreclosures. Despite recent interest, the economic implications remain murky. Contract terms are further complicated since the tenant may stay financially constrained, and not qualify for a mortgage, in the future. In this paper, we analyze the economics of a RTO contract under financial constraints in competitive equilibrium. We show that the equilibrium purchase price is extremely sensitive to the tenant’s financial constraints. This information is useful in negotiating contract terms for an interesting new development in the real estate industry.

Key words: Housing lease Option, Mortgage Requirement, Homeownership, Rent to Own, Credit tightening, Esscher Transform
1 Introduction

The number of subprime mortgages nearly doubled from 1.1 million in 2003 to 1.9 million in 2005. Near-prime Alt-A originations increased from 304,000 in 2003 to 1.1 million in 2005 — more than a three-fold increase in two years (Finance (2008)). Unsustainable easing of credit underwriting standards led to an increase in credit supply, which in turn, according to many, led to the Great Recession.\footnote{Securitization, which led to the loosening credit underwriting standards, led to such an incredible rise in credit supply for high risk mortgage borrowers (Demyanyk and Van Hemert (2011), Purnanandam (2011), Mian and Sufi (2009), Keys et al. (2010), Dell’Arriacca et al. (2008)).} After 2008, credit supply became scarce. More than 85% of the banks tightened their credit underwriting standards between 2008 - 2011 (Table 11 of the Comptroller of the Currency (2012) report).\footnote{A recent report by BankRegData shows that during 2011-2013, the volume of commercial and industrial loans outstanding increased by 27% whereas, despite a refinance boom, consumer mortgage credit increased by only 4%. On the regulation side, the consumer financial protection bureau (CFPB) was created in 2011 whose primary responsibility is to take a conservative approach to mortgage lending.}

Going forward, it appears unlikely that the credit standards will loosen anytime soon. For example, according to the “Origination Insight Report” published by Ellie Mae\footnote{The data approximates represents 3.5 MM (in 2013) mortgages that used Ellie Mae software.} — a mortgage origination software maker — the average FICO score for an approved loan is approximately 760, while the average FICO score of a denied loan is approximately 720. To put these numbers in perspective, before the Great Recession, a subprime mortgage borrower with a FICO score below 620 could easily qualify for a mortgage. Clearly, after the Recession, credit standards have tightened.

Noting the substantially tightened credit markets and the robust rental market\footnote{According to the data published by the Lincoln Institute of Land Policy, the average house price decreased from $303K to $198K between 2006 and 2012 — a decrease of 33%. On the other hand, the average rent did not decrease.} to the National Association of Homebuilders, Bernanke (2012) recommends the use of rent-to-own (RTO) housing contracts:

... Moreover, keeping paying tenants in homes–including leasing to the former owners at market rents–may, in some cases, be the best way to maintain property values and the quality of neighborhoods. REO\footnote{Homes owned by creditors are sometimes called real estate owned or REO properties.}-to-rental programs could potentially also minimize the amount of time that a vacant property languishes in REO inventory. That is, appropriately structured programs could help some involuntary...
renters become owners again by giving them options to purchase the homes they are renting.

This paper analyzes the economics of such RTO contracts.

As predicted by Bernanke, RTO contracts — a financial innovation — are increasingly getting popular. In a RTO contract, a Tenant or a Renter (also referred to as the Tenant-Buyer) and a Landlord (also referred to as the Landlord-Seller) enter into a lease agreement. The term of the lease is typically between 12 to 36 months. The lease period allows the Tenant-Buyer who under the current environment cannot meet the tougher loan qualification requirements to build up savings and creditworthiness. To summarize, RTO contracts are attractive to those who are financially constrained at the beginning but expect to be unconstrained by the end of the lease period. These contracts effectively force buyers to save. They also provide tenants the opportunity to evaluate a neighborhood and resolve income uncertainty before making a long-term financial commitment.

Despite the recent interest, economic implications of the RTO contract remains murky. For example, figuring out the purchase price or the lease period is a matter of pure guesswork for most buyers/sellers. To the best of our knowledge, Jaggia et al. (2014) are first to analyze a RTO contract using an option valuation framework. They analyze the economics by using the fact that the Landlord-Seller writes a call option to the Tenant-Buyer. In a frictionless framework, they value of the option in the same spirit as a typical financial call option.

However, a frictionless framework may not be appropriate. For example, in spite of the best efforts, due to variability in savings, the Tenant-Buyer may end up being financially constrained at the end of the lease term. In this case, Tenant-Buyer cannot go through with the purchase even though it may be optimal for her to do so. Therefore, due to financial constraints, RTO contract is not like a typical call option. Bernanke and government sponsored agencies also recognized the negative impact of financial constraints on the success of RTO contracts. In response, both Freddie Mac and Fannie Mae announced a pilot program to facilitate purchases stemming from RTO contracts in 2012 (Bernanke (2012)). In this paper, our main contribution

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6From analyzing Google trends, the search intensity of the word “RTO” increased from 33 in April, 2007 to 100 in February 2014 — a three-fold increase. Indeed typing the words “rent to own real estate” yields more than 200 books on Amazon.com.
is to analyze the effect of financial constraints on the RTO contract.

In competitive equilibrium, we derive a closed form expression of the call option with financial constraints. We use a mathematical technique known as an Esscher Transform to derive the closed form expression. Our set up is useful in understanding the link between the purchase price, the downpayment, the rental premium and the lease period in a competitive real estate market.

Our model consists of two periods, specifying the beginning and the end of the lease. In the first period, the Landlord-Seller and Tenant-Buyer negotiate the purchase price, the down payment and the monthly rent premium. The down payment and the monthly rent premium constitute the option premium. There are two sources of uncertainty: the house value and the Tenant-Buyer’s ability to secure a mortgage at the end of the second period. The Tenant-Buyer will exercise the option to purchase the house if and only if: (i) the house value in the second period is above the purchase price adjusted for the option premium and (ii) the Tenant-Buyer is not financially constrained.

The Landlord-Seller receives the option premium and in return she writes the call option (with strike price equal to the predetermined purchase price) to the Tenant-Buyer. The surplus accrued to the Landlord-Seller is equal to the option premium less the call option value. In a competitive real estate market, the implied purchase price is set so that the surplus accrued to the Landlord-Seller equals to zero.

We perform extensive comparative statics to evaluate the equilibrium purchase price of a hypothetical RTO contract. Specifically, we show that the equilibrium purchase price is extremely sensitive to financial constraints. We highlight cases where, for a financially constrained tenant, the equilibrium purchase price falls below the current market value of the house. Here, even at the reduced house price, the tenant will not be able to exercise the seemingly optimal option. In the absence of financial constraints, the equilibrium purchase price increases to the value predicted by the standard Black-Scholes model.

We also calibrate our model by analyzing the equilibrium purchase price of 20 major U.S. cities. There is a fair bit of variation in the purchase price across cities. It is the highest in Los Angeles, where the equilibrium purchase price on a $200K house is $226K, and the lowest
in Detroit where the equilibrium purchase is only $186K. We also compare the equilibrium purchase price computed with and without financial constraints. In general, the financial constraints decrease the negotiated premium by half. To preview our results, consider the case of San Francisco. If the Tenant-Buyer may be financially constrained in the future, the equilibrium purchase price is equal to $210K - a 5% premium. On the other hand, if the Tenant-Buyer will not be financially constrained in the future, the equilibrium purchase price is equal to $220K - a 10% premium.

The rest of the paper is as follows. In Section 2, we explain the background of a RTO contract. In this section, we explain the previous literature concerning bank lending standards and motivate the popularity behind RTO contracts. Section 3 introduces a simple two-period valuation model. Using Esscher Transform, we derive a closed form expression without making any distributional assumptions. Explicit valuation formula when the random variables are normally distributed are derived in Section 4. In this section, we also perform comparative statics and evaluate the equilibrium purchase price in 20 major U.S. cities. Lastly, in Section 5, we provide concluding remarks.

2 Background of RTO contracts

In this section, we motivate the underlying reasons behind the growing popularity of RTO contracts. We motivate in two parts. First, we review and provide new evidence of tightening credit standards in the housing market. Second, in light of the tighter lending standards, we show several examples of potential home buyers who are ideal candidates for a RTO contract.

2.1 Pro-cyclical bank lending standards

It is well documented that banks speed up lending activity during an economic expansion, which results in lower down payments and higher loan-to-value ratios (Gorton (2009) and Hardouvelis (2010)). Competition among banks magnifies the expansion of lending activity. Banks screen borrowers with less scrutiny due to competitive pressure during an economic expansion (Ruckes (2004), Dell’Ariccia and Marquez (2006), Hauswald and Marquez (2006), and Gorton and He
The increase in lending activity improves asset prices; increases collateral values; and in turn increases lending activity.

On the other hand, risk premia increase and the quantity of risk taking decreases during an economic contraction. Lang and Nakamura (1995) and Bernanke et al. (1996) document the “flight to liquidity phenomenon”: bank portfolios shift from high to low risk loans during a downturn. Simultaneously, non-performing loans rise, leading to lower capital reserves. In response, banks tighten credit standards. Bernanke et al. (1991) find that large negative shocks to bank capital lead to declines in bank lending. As a result, down payments increase and loan-to-value ratios decrease. This forms the classic (pro-cyclical) “lending cycle”. Berlin (2009) provides an excellent survey of the lending cycle literature.

2.2 New evidence of tightening credit standards

2.2.1 Credit tightening of conventional loans

One barometer of credit tightening stems from the analysis of the secondary mortgage market. Specifically, we analyze Fannie Mae’s portfolio. Fannie Mae issues minimum credit standards to mortgage lenders and the mortgages have to adhere to these standards in order for the mortgage lender to sell the mortgage to Fannie Mae. To this end, Fannie Mae issues a menu called “Eligibility Matrix” — a comprehensive set of (i) Loan to value ratios, (ii) FICO scores and (iii) maximum income to debt ratio — for conventional first mortgages to be eligible for delivery to Fannie Mae. The credit standards in the menu in turn govern the credit standards of mortgage lenders.

Figure 1 plots time series of the share of poor and good quality loans of Fannie Mae’s portfolio. The share of poor quality mortgages (loans with credit score below 620) decreased significantly between 2008-09 — this is consistent with Fannie Mae’s policy of tightening eligibility standards. The average FICO score of mortgages in Fannie Mae portfolio increased to 761 in 2009 from 738 in 2008 (2009 10-K). On the flip side, the share of good quality mortgages

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7Fannie Mae plays provides significant liquidity to the secondary mortgage market. The market share of Fannie Mae (according to 2013 10-K filing) was 40%. The estimated market share of new single-family mortgage-related issuance was 46% in the fourth quarter of 2013.
Figure 1: Plot of the FICO score of conventional conforming first time home buyers. The data is from the credit supplement provided by Fannie Mae.

(loans with credit score above 740) increased significantly after 2008-09. Note that there is no indication that the eligibility standards will revert back to the 2008 levels anytime soon.

2.2.2 Credit tightening of Federal Housing Agency (FHA) loans

Here, we analyze the portfolio share of Federal Housing Agency (FHA). The goal of FHA is to provide financing to borrowers who are typically financially constrained. After 2008, the share of home purchases financed with FHA mortgages went from 2 percent to over one-third of mortgages in the United States, as conventional mortgage lending dried up.\(^8\) Figure 2 plots time series of the share of poor and good quality loans of FHA portfolio. The evidence of credit tightening is clear. The share of poor quality loans (loans with FICO score below 620) decreases significantly after 2008 and the share of good quality loans (loans with FICO score above 720) increases significantly after 2008.

To summarize, the lending standards are pro-cyclical: the standards are lax during good times and the standards are strict during bad times. Fannie Mae and FHA’s share of good

\(^8\)FHA Wikipedia entry: \url{http://en.wikipedia.org/wiki/Federal_Housing_Administration}
quality loans increased after 2008, while the share of poor quality loans decreased. In other words, after 2008, it became even harder for financially constrained borrowers to get a loan. Next, we show the merits of a RTO contract in light of the tightened credit standards.

2.3 RTO contract motivation

A RTO contract provides home buyers the opportunity to evaluate a neighborhood and resolve income uncertainty before making a long term financial commitment. It is especially attractive to those who are financially constrained. To motivate the merit of a RTO contract, consider the following four scenarios:

1. James, an entrepreneur, declared bankruptcy a year ago due to a failed start up. Now, James has a stable job, and wants to purchase a house. He does not qualify for a conventional loan. Even FHA does not consider applications of borrowers who declared bankruptcy less than two years before the mortgage application. As a result, he is not able to meet the “time” guideline for a FHA loan.
Jill just graduated from college. She is currently working and doing well at her job: she expects promotion within a couple of years. Despite a promising future, she does not qualify for a conventional loan and does not meet the debt to income ratio requirement for a FHA loan.

John recently underwent a heart surgery and incurred large medical bills. In spite of having a good credit score and a stable job, he does not have enough excess wealth for a down payment. As a result, he does not meet the down payment requirement for either a conventional or a FHA loan.

Linda, a single mother, has recently been transferred to a small town in the Midwest. Although, she enjoys the relaxed lifestyle of a small town, she is not sure if this is the place where she would like to settle down.

These are some of the scenarios in which a RTO contracts are attractive to Tenant-Buyers. The first three examples concern potential home buyers who are financially constrained and do not qualify for mortgage financing. But, they expect not to be financially constrained in the future. James, with his stable job expects to repair his credit history; Jill with her promotion expects a significant rise in salary; John expects to save enough for a down payment. All the above borrowers have one need: they need time. The RTO contract will enable these borrowers to live in the home of their liking while giving them the option to stay there permanently. The term of the RTO contract is enough for them to become financially unconstrained. In scenario 4, Linda is not financially constrained but can benefit from the opportunity that RTO provides to evaluate a neighborhood and resolve income uncertainty before making a long-term financial commitment.

Alternatively and perhaps more importantly, lenders can use a RTO contract to renegotiate contracts with homeowners whose mortgages are underwater. Consider the following scenario:

John, a teacher, bought his home at the apex of the housing market for $300K. John entered into a 30 year fixed rate mortgage at 6% and was able to borrow 90% of the value of the house. After seven years, house price has dropped to $200K while the loan balance
is $241K. Since the mortgage is sufficiently underwater, John wants to default. Both the lender and John incur deadweight bankruptcy costs in case of default.\footnote{In Jaggia et al. (2014), both buyers and sellers face deadweight costs and they split the surplus using a Nash Bargaining mechanism.}

In the last scenario, the lender and John can enter into a RTO contract to avoid deadweight bankruptcy costs. For example, the lender can take over the ownership of the home while annuling the mortgage contract. Additionally, the lender can lease the home to John for three years and give John the option to buy back the home at a predetermined (negotiated) purchase price. To summarize, a RTO contract can potentially be a win-win situation for both the lender and John.

Next, we develop a theoretical framework to study the economics of a RTO contract.

### 3 A two-period model of a RTO contract

This section develops a two-period model to value a RTO contract under financial constraints. We follow Grenadier (1996) in our analysis of the financial constraints. In addition to rent, a RTO contract consists of several negotiable terms: a lease term $T$, a predetermined purchase price $\tilde{K}$, an upfront down payment and the monthly rent premium.\footnote{The contract also includes escrow procedures, amount and timing of broker commissions and other fees that are customary of a standard lease agreements. The agreement also specifies the timing of the home inspection, the party responsible for house repairs and maintenance and the party responsible for home insurance and property taxes. Lastly, the agreement also specifies the sequence of legal steps if the Tenant-Buyer is late in making the monthly payments.} Both the down payment and the rent premium count as a payment toward the purchase price. Since we consider a two-period model, we combine the down payment and monthly rent premium into one up-front payment — the payment is the “option premium” $OP$.

We divide this section in three parts. In the first part, we describe uncertainty. Specifically, we describe the joint evolution of the house price and the Tenant-Buyer’s excess wealth without making any distributional assumptions. In the second part, we value the call option in a RTO contract in closed form. In the last part, we solve for the purchase price in a perfectly competitive real estate market.
3.1 Sources of uncertainty

Consider a risk-neutral environment with two periods $t = \{0, T\}$, where $T$ is the length of time (in years) spanned by the two periods. Both players — the Tenant-Buyer and the Landlord-Seller — discount future cashflows at a constant rate $r$. There are two sources of uncertainty. First, the house value in the second period $T$ is random. We denote $H(t)$ to represent the value of the house in period $t$ and let

$$H(T) = H(0) \exp\{X\}.$$  

The random variable $X$, which can be interpreted as the continuously compounded rate of return over $T$ years, is governed by the cumulative probability distribution $F_X(x) = \Pr\{X \leq x\}$.

The second source of uncertainty concerns financial constraints. We denote $W(t)$ to represent a financial state variable (we define it later) of the buyer in period $t$ and let

$$W(T) = W(0) \exp\{Y\}.$$  

Again, the random variable $Y$ can be interpreted as the continuously compounded rate of return over $T$ years. The cumulative density function of $Y$ is denoted by $F_Y(y) = \Pr\{Y \leq y\}$ and the joint cumulative probability function of the random variables $X$ and $Y$ is denoted by $F_{XY}(x,y) = \Pr\{X \leq x \text{ and } Y \leq y\}$.

We model the financial constraints in the following manner. If $W(T)$ falls below an exogenous threshold $I$, we assume that the buyer will not be able to qualify for loan. This may occur because (i) the buyer is unable to make a requisite down payment required for a loan or (ii) the debt to income ratio of the buyer is insufficient to qualify for a loan. For clarity, we interpret $W(t)$ as the excess wealth of the Tenant-Buyer. The excess wealth is composed of both initial and future savings.
Lastly, let \( z = (z_1, z_2)' \) be a non-zero real vector such that the moment generating function

\[
M(z) = \mathbb{E}[\exp\{z_1X + z_2Y\}]
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\{z_1 x + z_2 y\} dF_{XY}(x, y)
\]  

exists. We will use the moment generating function in the valuation of the RTO contract, which is next.

### 3.2 Valuation of a RTO contract

The Tenant-Buyer agrees (after negotiation) to purchase from the Landlord-Seller at a purchase price \( \tilde{K} \) and in turn the Tenant-Buyer pays an option premium \( OP \) for the option. At the end of the second period \( T \), the Tenant-Buyer has the right but not the obligation to buy the house for an effective strike price \( K \): \( K \equiv \tilde{K} - OP \). Furthermore, the buyer will only exercise the option in the event (i) the call option is in the money \( \{H(T) \geq K\} \), and (ii) the Tenant-Buyer is not financially constrained \( \{W(T) \geq I\} \). With the definition,

\[
k_1 = \ln \left(\frac{K}{H(0)}\right) \quad \text{and} \quad k_2 = \ln \left(\frac{I}{W(0)}\right),
\]

let

\[
\mathcal{A} = \{X \geq k_1, Y \geq k_2\}
\]

represent the event in which the Tenant-Buyer will exercise the option. The expected present value of the call option, denoted by \( C \), is

\[
C \equiv C(H(0), W(0)) = \exp\{-rT\} \int_{k_2}^{\infty} \int_{k_1}^{\infty} dF_{XY}(x, y) (H(0) \exp\{x\} - K) \\
= \exp\{-rT\} \mathbb{E}[\{H(T) - K\} \mathcal{I}(\mathcal{A})] \\
= \exp\{-rT\} \mathbb{E}[H(T) \mathcal{I}(\mathcal{A})] - \exp\{-rT\} K \mathbb{E}[\mathcal{I}(\mathcal{A})] \\
= C_1 - C_2,
\]
where $\mathcal{I}(\cdot)$ denotes the indicator function of an event.

Standard calculations yield that

$$C_2 = \exp\{-rT\} K \left[ 1 - F_X(k_1) - F_Y(k_2) + F_{XY}(k_1, k_2) \right]. \quad (5)$$

The expression for $C_1$ is more complicated. In order to get a closed form solution, we use the concept of Esscher Transform with parameter $z$. Specifically, dividing equation (1) by $M(z)$, we see that

$$1 = \mathbb{E}[\exp\{z_1X + z_2Y\ }] / M(z)$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dF_{XY}(x, y; z), \quad (6)$$

where we define

$$dF_{XY}(x, y; z) \equiv \exp\{z_1x + z_2y\} dF_{XY}(x, y) / M(z).$$

Upon inspection of equation (6), the probability measure, $dF_{XY}(x, y; z)$, of the joint random variables $X$ and $Y$ has been modified. We say that the modified distribution, $dF_{XY}(x, y; z)$, is the Esscher Transform with parameter $z$ of the original distribution $dF_{XY}(x, y)$. Because the exponential function is positive, the modified probability measure is equivalent to the original probability measure; that is, both probability measures have the same null sets (sets of probability measure zero).

In order to identify the new probability measure, it is convenient to analyze the moment generating function under the new measure. Specifically,

$$M(u; z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\{u_1x + u_2y\} dF_{XY}(x, y; z)$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\{u_1x + u_2y\} \exp\{z_1x + z_2y\} dF_{XY}(x, y) / M(z)$$

$$= M(u + z) / M(z). \quad (7)$$
Defining $\mathbf{1}_1 = (1, 0)'$, the expression for $C_1$ can be written as

$$
C_1 = \exp\{-rT\} \mathbb{E}[H(T) \mathcal{I}(\mathcal{A})] \\
= \exp\{-rT\} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H(0) \exp\{X\} \mathcal{I}(\mathcal{A}) \, dF_{XY}(x, y) \\
= \exp\{-rT\} H(0) M(\mathbf{1}_1) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{I}(\mathcal{A}) \, dF_{XY}(x, y; \mathbf{1}_1) \\
= \exp\{-rT\} H(0) M(\mathbf{1}_1) \Pr\{X \geq k_1, Y \geq k_2; \mathbf{1}_1\}. \quad (8)
$$

Using the same technique as before, we have that

$$
C_1 = \exp\{-rT\} H_0 M(\mathbf{1}_1) \times [1 - F_X(k_1; \mathbf{1}_1) - F_Y(k_2; \mathbf{1}_1) + F_{XY}(k_1, k_2; \mathbf{1}_1)]. \quad (9)
$$

We summarize the call option value in the following proposition.

**Proposition 1** The call option $C$ associated with the RTO contract is worth $C = C_1 - C_2$, where

$$
C_2 = \exp\{-rT\} K [1 - F_X(k_1) - F_Y(k_2) + F_{XY}(k_1, k_2)],
$$

and

$$
C_1 = \exp\{-rT\} H_0 M(\mathbf{1}_1) \times [1 - F_X(k_1; \mathbf{1}_1) - F_Y(k_2; \mathbf{1}_1) + F_{XY}(k_1, k_2; \mathbf{1}_1)],
$$

where the new probability measure is defined by equation (6).

In order to get a better understanding of the call option value, consider two special cases.

**Case 1: No Financial Constraints** — In the event that the Tenant-Buyer will not be financially constrained in the future for sure, $Y \downarrow 0$ and hence $k_2 \downarrow -\infty$. In this case, equation (2) becomes

$$
C_{BS} \equiv \exp\{-rT\} \int_{-\infty}^{\infty} dF_X(x) \left(H(0) \exp\{x\} - K\right) \geq C. \quad (10)
$$

We denoted the call option value without financial constraints as $C_{BS}$ to highlight the fact that this condition is isomorphic to the Black Scholes call option. As a matter of fact, in the numerical example, where we assume that $X$ and $Y$ are jointly normally distributed, the call
option formula reduces to the Black-Scholes formula when there are no financial constraints. Upon inspection, it is also clear that $C_{BS} \geq C$. We summarize the result in equation (10) in the following proposition:

**Proposition 2** The value of the call option with financial constraints is less than the value of the call option without financial constraints: $C \leq C_{BS}$. Furthermore, $C \uparrow C_{BS}$ as the financial constraints relax.

**Case 2: Impact of Financial constraints when $X$ and $Y$ are independent** — When house value and Tenant-Buyer’s excess wealth are independent, equation (2) becomes

\[ C = C_{BS} \Pr\{Y \leq k_2\}. \]  

Equation (11) shows the dramatic effect of the financial constraints. If the Tenant-Buyer is unlikely to get a mortgage, then equation (11) says that the call option is effectively worth less. We summarize the implications of equation (11) in the following proposition.

**Proposition 3** In the case where $X$ and $Y$ are independent, the call option value is strictly lower than the Black-Scholes value: $C \leq C_{BS}$. More importantly, the call option is inversely proportional to the likelihood of being financially constrained.

The implication of equation (11) is significant. In the limit case where the Tenant-Buyer will be financially constrained in the future, the call option value is worthless. In this sense, the prospect of being financial constrained in the future has a significant impact on the success of a RTO contract.

To summarize, with equations (5) and (9), we derive a closed form expression for the call option. We have done some without making any joint distributional assumptions regarding the house value or the excess wealth. We use the technique of Esscher Transform to get the closed form expression. Note that the Esscher Transform methodology requires no double integrals to be evaluated! Next, we analyze the equilibrium purchase price.
3.3 Purchase Price

The surplus accrued to the Landlord-Seller is

\[ S(\tilde{K}) = OP - C(\tilde{K}). \tag{12} \]

We write \( C \equiv C(\tilde{K}) \) to explicitly highlight that the call price \( C \) is a function of the purchase price \( \tilde{K} \). Landlord-Seller receives the option premium in the first period \( t = 0 \) and in turn the landlord sells a call option. In a perfectly competitive real estate RTO market, the purchase price is set so that the surplus accrued to the Landlord decreases to zero. Formally, the competitive purchase price \( K^* \) is the implicit solution to

\[ K^* = \{ \tilde{K} : S(\tilde{K}) = 0 \}. \tag{13} \]

Next, we give a numerical example. We assume that both \( X \) and \( Y \) are bivariate normal random variables. With this assumption, we give an explicit expression for the call option.

4 RTO contract with bivariate normal random variables

Let \( X = (X,Y)' \) is a bivariate normal random variable. Let

\[
\mu \equiv \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} \mu_H - \frac{1}{2}\sigma_H^2 \\ \mu_W - \frac{1}{2}\sigma_W^2 \end{bmatrix}, \quad \text{and} \quad \Sigma \equiv \begin{bmatrix} \sigma_1^2 & \rho \sigma_1 \sigma_2 \\ \rho \sigma_1 \sigma_2 & \sigma_2^2 \end{bmatrix} = \begin{bmatrix} \sigma_H^2 & \rho \sigma_H \sigma_W \\ \rho \sigma_H \sigma_W & \sigma_W^2 \end{bmatrix}
\]

denote the mean vector and covariance matrix of \( X \). Standard calculation shows that the moment generating function is

\[ M(z) = \exp\{z'\mu + \frac{1}{2}z'\Sigma z\} \]

Slight algebra shows that the moment generating function in the Esscher measure with
parameter \( z \) is

\[
M(u; z) = \frac{M(u + z)}{M(z)} = \exp\{u'\tilde{\mu} + \frac{1}{2}u'\Sigma u\}, \quad \text{where}
\]

\[
\tilde{\mu} \equiv (\tilde{\mu}_1, \tilde{\mu}_2)' = \mu + \Sigma z.
\]  

(14)

Upon inspection of the equation (14), the new probability measure is also bivariate normal with a modified mean \( \tilde{\mu} \) but with the same covariance \( \Sigma \).

Let \( \Phi(a) \) and \( \Phi_2(a, b; \rho) \) denote the univariate and bivariate standard normal cumulative density function. Substituting \( 1_1 \) for \( z \) in equation (14), the RTO value is

\[
C = C_1 - C_2
\]

where

\[
C_2 = \exp\{-rT\} K \left[ 1 - \Phi\left(\frac{k_1 - \mu_1}{\sigma_1}\right) - \Phi\left(\frac{k_2 - \mu_2}{\sigma_2}\right) + \Phi_2\left(\frac{k_1 - \mu_1}{\sigma_1}, \frac{k_2 - \mu_2}{\sigma_2}, \rho\right) \right],
\]

and

\[
C_1 = \exp\{-rT\} H_0 M(1_1) \left[ 1 - \Phi\left(\frac{k_1 - \tilde{\mu}_1}{\sigma_1}\right) - \Phi\left(\frac{k_2 - \tilde{\mu}_2}{\sigma_2}\right) + \Phi_2\left(\frac{k_1 - \tilde{\mu}_1}{\sigma_1}, \frac{k_2 - \tilde{\mu}_2}{\sigma_2}, \rho\right) \right].
\]

### 4.1 Model calibration

In calibrating the model, we first choose the house value \( H(0) \) to equal $200,000. This value is on the same order of magnitude as the Case Shiller average house price. We choose the house growth rate \( \mu_h \) to be 2.50% and house volatility rate \( \sigma_h \) to be 5.00% — these parameters are from the analysis of 20 largest U.S. cities. Consistent with a typical RTO contract, we assume RTO contract term \( T \) to be 3 years. Following Grenadier (1996), we assume that the discount rate \( r \) of both agents is 4.00%. We also choose the option premium \( OP \) to be 6.00% of the house value \( H(0) \). The option premium approximately equals a 2.50% down payment and the
present value of a $200 monthly rent premium, which are typical terms of a RTO contract.

Contract terms are negotiated privately between the Tenant-Buyer and the Landlord-Seller. Both parties will perform their due diligence. For instance, both parties will show germane paperwork that shows their creditworthiness or labor income. In this sense, in addition to the above mentioned parameters, calibration of the model requires estimates of the financial constraint measure $W(0)/I$ along with excess wealth growth rate $\mu_W$, excess wealth volatility $\sigma_W$, excess wealth and house value correlation $\rho$. As mentioned earlier, we interpret $W(t)$ as the excess wealth of the Tenant-Buyer composed of initial and future savings and $W(0)$ represents the present value of excess wealth.

For expositional clarity, consider John’s case. Recall that due to medical expenses, John did not have enough for a downpayment even though he had a good credit score and expects to build up significant savings. We assume that John places his savings in a mutual fund say Fidelity Balanced Fund (FBALX). Consistent with FBALX’s returns, we choose excess wealth growth rate $\mu_W$ to be 5.00%, excess wealth volatility $\sigma_W$ to be 10.00% and excess wealth and house value correlation $\rho$ to be 15.00%.

We define the financial constraint measure $W(0)/I$ as follows. As mentioned earlier, RTO contracts are attractive to those who are financially constrained in the first period but expect to be financially unconstrained in the second period. Therefore, given any threshold $I$, the Tenant-Buyer expects $W(T)/I \geq 1$. Arguably, this condition is satisfied if the Tenant-Buyer has made significant savings in the second period to qualify for a mortgage. Regardless of how we define the threshold, it is reasonable to assume that the lower limit of $W(0)/I$ is the ratio of its time 0 and time $T$ values, adjusted for the option premium. We approximate the lower limit of $W(0) = I$ by the present value factor $\exp\{-rT\}$ which, for $r = 4\%$ and $T = 3$ years, equals 0.89; we set the base case value as 0.90. Table 1 summarizes our base case values used for model calibration.

\footnote{We also performed our analysis using NIPA data and the results remain qualitatively and quantitatively similar.}
Table 1: This table shows the base case parameter values.

<table>
<thead>
<tr>
<th>RTO contract variables</th>
<th>Variable</th>
<th>Symbol</th>
<th>Values</th>
<th>Variable</th>
<th>Symbol</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>House value</td>
<td>$H(0)$</td>
<td>$200K$</td>
<td></td>
<td>Option premium</td>
<td>$OP / H(0)$</td>
<td>6.00% (% of house value)</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$r$</td>
<td>4.00%</td>
<td></td>
<td>Wealth growth rate</td>
<td>$\mu_W$</td>
<td>5.00%</td>
</tr>
<tr>
<td>RTO term length</td>
<td>$T$</td>
<td>3 years</td>
<td></td>
<td>Wealth volatility rate</td>
<td>$\sigma_W$</td>
<td>10.00%</td>
</tr>
<tr>
<td>Financial constraint measure</td>
<td>$W(0) / I$</td>
<td>0.90</td>
<td></td>
<td>House growth rate</td>
<td>$\mu_h$</td>
<td>2.50%</td>
</tr>
<tr>
<td>Correlation</td>
<td>$\rho$</td>
<td>0.15</td>
<td></td>
<td>House volatility rate</td>
<td>$\sigma_h$</td>
<td>4.00%</td>
</tr>
</tbody>
</table>

4.2 Comparative statics of the implied purchase price

4.2.1 Effect of correlation on the implied purchase price

Figure 3 plots the implied purchase price ($K^*$) as a function of the financial constraint measure $W(0) / I$ for three levels of correlation: $\rho = 35.00\%$ (the high case), $\rho = 15.00\%$ (the base case) and $\rho = -5.00\%$ (the low case).

Note that the implied purchase price increases as correlation increases, albeit marginally. This result can be understood better in the limit case of $\rho = 100.00\%$. In the limit case, a high house value in time $T$, $H(T)$ corresponds perfectly to a high net worth in period $T$, $W(T)$. That is, the event that the option in the money, is also the event that the Tenant-Buyer is not financially constrained. Therefore, the call option value and in turn the implied purchase price increases with correlation.

At the same time, the financial constraint has a greater impact on the implied purchase price at lower correlation values. This is because of the divergence of the event that the option is in the money and that the Tenant-Buyer is not financially constrained.
4.2.2 Effect of house growth rate and house volatility rate on the implied purchase price

Figure 4 plots the implied purchase price \( (K^*) \) as a function of the financial constraint measure \( W(0)/I \) for three levels of house value growth rate: \( \mu_h = 5.00\% \) (the high case), \( \mu_h = 2.50\% \) (the base case) and \( \mu_h = 0.00\% \) (the low case). An increase in the growth rate implies that the house is more likely to be valuable in the future. Then, as expected, the call value increases with the house value growth rate, which in turn increases the implied purchase price.

Figure 5 plots the implied purchase price \( (K^*) \) as a function of the financial constraint measure \( W(0)/I \) for three levels of house value volatility: \( \sigma_h = 6.00\% \) (the high case), \( \sigma_h = 4.00\% \) (the base case) and \( \sigma_h = 2.00\% \) (the low case). As expected, the call value and in turn the implied purchase price increase with the house value volatility.
Implied Purchase Price $K^*$ / $H(0)$

Measure of financial constraint: $W(0) / I$

High case $h = 5.00\%$

Base case $h = 2.50\%$

Low case $h = 0.00\%$

Figure 4: Plot of the implied purchase price ($K^*$) versus financial constraint measure $W(0) / I$ for different house value growth rate $\mu_h$

Implied Purchase Price $K^*$ / $H(0)$

Measure of financial constraint: $W(0) / I$

High case $\sigma_h = 6.00\%$

Base case $\sigma_h = 4.00\%$

Low case $\sigma_h = 2.00\%$

Figure 5: Plot of the implied purchase price ($K^*$) versus financial constraint measure $W(0) / I$ for different house value volatility $\sigma_h$
4.2.3 Effect of excess wealth growth rate and excess wealth volatility on the implied purchase price

Figure 6 plots the implied purchase price ($K^*$) as a function of the financial constraint measure $W(0)/I$ for three levels of excess wealth growth rate: $\mu_w = 7.50\%$ (the high case), $\mu_w = 5.00\%$ (the base case) and $\mu_w = 2.50\%$ (the low case). An increase in the growth rate implies that the Tenant-Buyer is less likely to be financially constrained in the future. Then, as expected, the call value and in turn the implied purchase price increase with the excess wealth growth rate. Note that effect of the growth rate can be quite dramatic. In the low case, for low levels of $W(0)/I$, the Tenant-Buyer will remain financially constrained with high probability. As a result the call option approaches zero rapidly. Therefore the purchase price also decreases rapidly as indicated by the sharp drop (Low case: solid line with markers).

Figure 7 plots the implied purchase price ($K^*$) as a function of the financial constraint measure $W(0)/I$ for three levels of excess wealth volatility: $\sigma_w = 15.00\%$ (the high case), $\sigma_w = 10.00\%$ (the base case) and $\sigma_w = 5.00\%$ (the low case). The comparative statics are dramatic and somewhat surprising. For high levels of the financial constraint measure $W(0)/I$, the call option decreases with the excess wealth volatility. The intuition is as follows. For high levels of the financial constraint measure $W(0)/I$, the Tenant-Buyer is unlikely to be financially constrained in the future. However, as the excess wealth volatility increases, the likelihood of negative shocks to the excess wealth increases. This increase in likelihood in turn lowers the chance that the Tenant-Buyer will be financially unconstrained in the second period. In this manner, the call option value and in turn the implied purchase price decrease with an increase in the excess wealth volatility.

On the other hand, for low levels of the financial constraint measure $W(0)/I$, the Tenant-Buyer is likely to be financially constrained in the future. The call value is close to zero in this case anyway. An increase in volatility increases the likelihood of positive shocks to the net worth, which in turn increases the likelihood of exercise. Therefore, the option value and in turn the implied purchase price increase with the excess wealth volatility for low levels of the financial constraint measure $W(0)/I$. 
Figure 6: Plot of the implied purchase price ($K^*$) versus financial constraint measure $W(0)/I$ for different wealth growth rates $\mu_w$.

### 4.2.4 Effect of Option premium on the implied purchase price

Figure 8 plots the implied purchase price ($K^*$) as a function of the financial constraint measure $W(0)/I$ for three levels of option premium: $OP/H(0) = 8.00\%$ (the high case), $OP/H(0) = 6.00\%$ (the base case) and $OP/H(0) = 4.00\%$ (the low case). The comparative statics are similar to the previous case. For high levels of the financial constraint measure $W(0)/I$, the Tenant-Buyer will most likely not be financially constrained in the future. Therefore, an increase in the option premium decreases the implied purchase price.

On the other hand, for low levels of the financial constraint measure $W(0)/I$, the Tenant-Buyer is likely to be financially constrained in the future. For clarity, fix a candidate purchase price $K_1 \approx H(0)$. Due to financial constraints, the call value is effectively zero. Here a dollar increase in the option premium increases the surplus by approximately a dollar as the call
value does not change. Therefore, for the surplus to decrease to zero, the purchase price has to decrease significantly. Formally, the purchase price $K^* \ll K_1$.

4.3 Comparison of the implied purchase price across different cities

Table 2 calculates the implied purchase price $K^*$ for different cities. Upon inspection, there is quite a bit of variation across cities. For instance, Los Angeles has the fastest house growth at 5.27% and Detroit has the slowest growth at -0.84%. Additionally, Las Vegas is the most volatile with volatility being 6.41% and Denver is the least volatile with volatility being 1.73%. All the cities are positively correlated with the stock market index FBALX: Denver is the most correlated and somewhat unexpectedly New York is the least correlated. Due to the variation in the house price dynamics across cities, the implied purchase prices (with and without constraints) also vary across cities. Consider Las Vegas for example. Las Vegas had one of the highest foreclosure rates in the nation. The house values grew at an annual rate of 1.72%
with a volatility of 6.41%. Additionally, the house price growth rate is not too correlated with FBALX (the correlation coefficient is 13.61%). With these city specific parameters, the implied purchase price without constraints is $214K — a 7% premium. However, with constraints, the implied purchase price decreases to $202K — a mere 1% premium. That is, financial constraints play a pivotal role in the determining the equilibrium purchase price.

5 Conclusion

The Great recession caused millions of foreclosures leading to sharp decline in house prices. Stricter lending standards further reduced housing demand. Interestingly, the rental market remained robust during the crisis and post-crisis periods. Noting the strong rental market, ex-chairman Bernanke effectively suggested the use of RTO housing contracts to stem foreclosures.

In this paper, we develop a theoretical framework to study the economics of a RTO contract
Table 2: This table shows the implied purchase price across 20 largest U.S. cities using base case value for the remaining parameters. The data are from Case Shiller city index.

<table>
<thead>
<tr>
<th>City</th>
<th>House growth rate $\mu_h$</th>
<th>House volatility rate $\sigma_h$</th>
<th>Correlation with FBALX $\rho$</th>
<th>Implied Purchase Price $K^*$ with Constraints</th>
<th>Implied Purchase Price $K^*$ without Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-City</td>
<td>3.63%</td>
<td>3.49%</td>
<td>16.83%</td>
<td>1.06</td>
<td>1.11</td>
</tr>
<tr>
<td>20-City</td>
<td>3.13%</td>
<td>3.28%</td>
<td>18.13%</td>
<td>1.05</td>
<td>1.10</td>
</tr>
<tr>
<td>Atlanta</td>
<td>0.67%</td>
<td>3.45%</td>
<td>16.28%</td>
<td>0.97</td>
<td>1.02</td>
</tr>
<tr>
<td>Boston</td>
<td>2.93%</td>
<td>2.38%</td>
<td>8.08%</td>
<td>1.04</td>
<td>1.09</td>
</tr>
<tr>
<td>Charlotte</td>
<td>1.54%</td>
<td>1.96%</td>
<td>14.64%</td>
<td>0.99</td>
<td>1.04</td>
</tr>
<tr>
<td>Chicago</td>
<td>1.21%</td>
<td>3.11%</td>
<td>16.55%</td>
<td>0.98</td>
<td>1.03</td>
</tr>
<tr>
<td>Cleveland</td>
<td>0.19%</td>
<td>2.63%</td>
<td>17.23%</td>
<td>0.95</td>
<td>1.00</td>
</tr>
<tr>
<td>Dallas</td>
<td>1.80%</td>
<td>1.82%</td>
<td>20.60%</td>
<td>1.00</td>
<td>1.05</td>
</tr>
<tr>
<td>Denver</td>
<td>1.95%</td>
<td>1.73%</td>
<td>27.14%</td>
<td>1.01</td>
<td>1.05</td>
</tr>
<tr>
<td>Detroit</td>
<td>-0.84%</td>
<td>4.21%</td>
<td>18.67%</td>
<td>0.93</td>
<td>0.97</td>
</tr>
<tr>
<td>Las Vegas</td>
<td>1.72%</td>
<td>6.41%</td>
<td>13.61%</td>
<td>1.01</td>
<td>1.07</td>
</tr>
<tr>
<td>Los Angeles</td>
<td>5.27%</td>
<td>4.87%</td>
<td>18.68%</td>
<td>1.13</td>
<td>1.18</td>
</tr>
<tr>
<td>Miami</td>
<td>3.93%</td>
<td>5.26%</td>
<td>16.67%</td>
<td>1.08</td>
<td>1.14</td>
</tr>
<tr>
<td>Minneapolis</td>
<td>1.68%</td>
<td>3.97%</td>
<td>14.60%</td>
<td>1.00</td>
<td>1.05</td>
</tr>
<tr>
<td>New York</td>
<td>3.40%</td>
<td>2.74%</td>
<td>1.47%</td>
<td>1.05</td>
<td>1.10</td>
</tr>
<tr>
<td>Phoenix</td>
<td>2.59%</td>
<td>6.14%</td>
<td>21.81%</td>
<td>1.05</td>
<td>1.10</td>
</tr>
<tr>
<td>Portland</td>
<td>3.38%</td>
<td>3.14%</td>
<td>13.33%</td>
<td>1.05</td>
<td>1.10</td>
</tr>
<tr>
<td>San Diego</td>
<td>4.03%</td>
<td>4.70%</td>
<td>18.42%</td>
<td>1.08</td>
<td>1.14</td>
</tr>
<tr>
<td>San Francisco</td>
<td>2.72%</td>
<td>5.23%</td>
<td>26.06%</td>
<td>1.05</td>
<td>1.10</td>
</tr>
<tr>
<td>Seattle</td>
<td>3.17%</td>
<td>3.00%</td>
<td>16.79%</td>
<td>1.05</td>
<td>1.10</td>
</tr>
<tr>
<td>Tampa</td>
<td>2.83%</td>
<td>4.47%</td>
<td>19.55%</td>
<td>1.04</td>
<td>1.09</td>
</tr>
<tr>
<td>Washington DC</td>
<td>4.73%</td>
<td>3.79%</td>
<td>19.65%</td>
<td>1.11</td>
<td>1.15</td>
</tr>
</tbody>
</table>
in the presence of financial constraints faced by the buyer. We perform extensive comparative statics to evaluate the equilibrium purchase price of a hypothetical RTO contract. Specifically, we show that the equilibrium purchase price is extremely sensitive to financial constraints. We also calibrate our model by analyzing the equilibrium purchase price of 20 major U.S. cities. There is a fair bit of variation in the purchase price across cities. We also compare the equilibrium purchase price computed with and without financial constraints. In general, the financial constraints decrease the negotiated premium by half. Naysayers will insist on a moral hazard problem with buyers mimicking financial distress in order to negotiate favorable contract terms. However, one would expect the Landlord-Seller to perform due diligence in analyzing the financial constraints of the Tenant-Buyer. It is in their interest for both parties to show germane paperwork that shows their creditworthiness.

After our analysis, one point is clear: RTO contracts are not a “silver bullet”. When the Tenants constraints are severe, the call option is worthless to the buyer and as a result, equilibrium purchase price declines towards zero. In other words, negotiations between the landlord and the tenant breakdown. On the other hand, when the financial constraints are not so severe, RTO contracts serve their intended purpose. They provide homebuyers the opportunity to evaluate a neighborhood and resolve income uncertainty before making a long-term financial commitment.

With a RTO contract, homeowners who are behind in their mortgages can still stay in their home. For example consider a homeowner whose mortgage is underwater. Rather than evicting, the lender can consider entering into a RTO contract with the homeowner. Contractually, lender takes over the deed and in return enters into a three-year RTO contract at a negotiated purchase price. In the absence of RTO, if the owner defaults, the lender suffers significant deadweight costs: eviction cost, legal cost, marketing cost and other administrative costs. From the owners perspective, default option also has deadweight costs: she may not qualify for a mortgage for a considerable period; she suffers social shame, she has to uproot family, and she incurs significant moving expense. A RTO contract, the deadweight costs suffered by the lender and the owner are substantially reduced. Therefore, in the spirit of Bernanke (2012), a RTO contract can be a win-win situation for the lender as well as the homeowner.
References


