Introduction to Futures and Options

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Spring 2014 — Lecture note on Forwards
**Forward Contracts**

**Definition:** A forward contract is a commitment to purchase at a future date a given amount of a commodity or an asset at a price agreed on today.

<table>
<thead>
<tr>
<th>Agreement Date</th>
<th>Settlement Date</th>
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<td>T</td>
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Buyer and Seller meet
Forward Contract Example

Mustang Inc enters into a forward contract with SLO Credit Union.

Jargon:
Mustang Inc is said to be short the Rupee per dollar exchange rate. That is Mustang Inc will lose money if Rupee depreciates relative to the dollar. Similarly, SLO Credit Union is long the Rupee per dollar exchange rate.
Forward Contract features

1. **Forward contracts are traded over the counter** — Someone from Mustang Inc literally gave a call to some one at SLO Credit Union. Buyer and Seller have to meet.

2. **Forward contracts are custom tailored** — Suppose Mustang Inc wanted to hedge $\sqrt{2} \times 10^6$ dollars worth. This is an irrational number but because two agents talk on the phone, then can agree to such a contract. For example, they can customize
   
   - What state to sue each other if there is a dispute?
   - How do you terminate the contract prior to expiration?
   - How do margin call each other?
   - How do you deliver the final rupees or dollars?
   - Precise scenarios that talk about the delivery of the asset.
3. Forward contracts have significant counter party risk — Suppose the Rupee depreciated significantly. In this case, SLO Credit Union owes a substantial amount to Mustang Inc. However, SLO Credit Union can declare bankruptcy and not pay Mustang Inc, in which case Mustang Inc will be really screwed. It lost money on the hedge (even though it is supposed to make money) and it receives a lower value for its services. Therefore, in the contract

- there will be a clause that explicitly points out what to do in such a situation
- there will be some language regarding collateral. For instance, both counterparties would post some collateral which will be safeguarded by a Trustee.

4. No money changes hands at time 0 in a forward contract — Money changes hands only on the settlement date.
Forward Contract terminology

- **Underlying Asset**— Commodity or asset that is being delivered i.e. Corn, gold, S&P 500 etc. In our example, it is the exchange rate. SLO Credit Union can deliver dollars (cash) and Mustang Inc. delivers Rupees (cash).

- **Deliver or Expiration Date**— Agreed on delivery date for exchange of the underlying asset and cash or cash settlement.

- **Forward Price**— Agreed on price for payment on the delivery date

- **Spot Price**— Price for immediate delivery of the underlying asset. In our example, the spot exchange rate is 60 rupees per dollar
Payoff from the Forward Contracts

In general, if $K$ is the delivery price and $S_T$ is the spot price of the asset at maturity $T$, then

the party that is long the forward contract receives

$$S_T - K.$$ 

The payoff for a short forward contract is

$$K - S_T.$$
Second example

Scenario— Party A agrees to buy a $1,000 face value, 90-day Treasury bill from Party B 30 days from now at a price $990.

1. Who is long?

2. Who is short?

3. What is the forward price?

4. What is the underlying commodity?

5. What is the time to maturity?
Suppose 30 days from now, T-bills are trading at $992, then the short must deliver the T-bill to the long in exchange for a $990 payment.

The contract can be settled several ways — **this needs to be specified in the contract**. Usually, there are two options:

- **Deliverable Forward Contract** — The short delivers the actual instrument, in this case a $1,000 face value, 90-day T-bill.

- **Cash Settlement** — The party that has a position with negative value is obligated to pay that amount to the other party. Example: the short pays $2 to the long.

**What happens if the short does not deliver?**
Terminating the forward contract prior to expiration

Suppose ten days after inception (it was originally a 30-day contract), the 20-day forward price of a $1,000 face value, 90-day T-bill is $993. The short, expecting the price to be even higher by the delivery date, wishes to terminate the contract.

What is the counterparty risk faced by all parties?
Players in the forward contract

- Government, corporations or non-profit institutions usually enter into forward contracts to hedge — they prefer to lock in a price.

- Dealers act as middlemen — they match parties that want to go long or short. The dealers will also enter contracts with other dealers.

**Jargon**— The dealer’s quote desk will quote a buying price at which they will go long and a slightly higher price at which they will go short. The bid/ask spread between the two is dealer’s compensation for administrative costs as well as credit risk.
What is the forward price?

The forward price is determined by the notion of Arbitrage. More importantly, it means that

\[ \downarrow \]

Your opinion or my opinion does not matter

**Arbitrage** means that if future cashflows generated by two portfolios is the same, then the two portfolios should have the same price.

Otherwise, market participants can short the expensive portfolio and go long the cheaper one and make free money.
A non-financial example of arbitrage

Suppose that an iPhone is selling for $800 in the US and for £500 in the UK.

Assume that the current exchange rate is £1 = $2. A simple conversion will tell us that an iPhone is worth more in the UK.

With the presence of such mis-pricing, an investor can seek to take advantage of such a situation by adopting the following strategy:

1. Buy an iPhone in the US for $800

2. Sell it in the UK for £500

3. Convert £500 into $1,000.
Therefore, if one were to follow this strategy for 500 iPhones, the profit would be a whopping $500 \times $200 = $100,000.

If you follow this, then your friend will follow this, and everyone will get rich. Market forces will then either

1. increase demand of iphone in US which will increase its price

2. decrease demand for UK iphone which will decrease its price in UK

*It is important to note that this example is very simplistic as it ignores several other factors like exchange rate risk and transaction costs. However, the principle is sound: When there is a difference in pricing in 2 or more markets, investors attempt to buy in the cheaper market and sell in the expensive market.*
A financial example of arbitrage

Environment: Suppose you can either borrow or lend in UK at an interest rate of 12% ($r_{UK}$).

Suppose you can either borrow or lend in US at an interest rate of 10% ($r_{US}$).

Lastly, suppose the current exchange rate is £1 = $1.50 ($S_0$) and the 1 year forward price is £1 = $1.55 ($F$), then there is Arbitrage

How?

Notation: The spot exchange rate in a year from now is denoted by $S_T$
A financial example of arbitrage — Contd

At time zero,

1. Borrow $1.50 in US. You will have to pay an interest rate of 10% (Trade 1)

2. Convert it into one pound. Lend that pound so that it earns 12% interest (Trade 2)

3. Sell forward 1.12 pounds forward at a price of $1.55. This means that a year from now you will get $1.736 and you will have to deliver 1.12 pounds (Trade 3)
In one year (all the trades unwind),

1. You receive 1.12 pounds \(\text{(Trade 2)}\)

2. You deliver 1.12 pounds and get $1.736 in return \(\text{(Trade 3)}\)

3. You have to pay $1.65 = $1.50 \times 1.10 \(\text{(Trade 1)}\)

4. You pocket $0.086 = $1.736 - $1.65
### Forward Spot Parity

<table>
<thead>
<tr>
<th>Portfolio 1</th>
<th>Portfolio 2</th>
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<tbody>
<tr>
<td><strong>Trade description</strong></td>
<td><strong>Borrow $x$ pounds in UK</strong></td>
</tr>
<tr>
<td>time equals zero</td>
<td>£$x$</td>
</tr>
<tr>
<td><strong>Sell 1 pound forward</strong></td>
<td>- $x(1 + r_{UK})$</td>
</tr>
<tr>
<td>You deliver 1 £ worth $S_T$ dollars</td>
<td>You pay $r_{UK}$ in interest</td>
</tr>
<tr>
<td>You receive $F$ dollars</td>
<td></td>
</tr>
<tr>
<td>time equals one year</td>
<td>$x \times S_0 \times (1 + r_{US})$</td>
</tr>
<tr>
<td></td>
<td>You receive $r_{US}$ in interest</td>
</tr>
</tbody>
</table>

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Spring 2014
Forward Spot Parity — Summary

Set  

\[ x = \frac{1}{1 + r_{UK}} \]

- You have to pay out £1 from Portfolio 2. This is worth $S_T$ dollars. Also, in portfolio 1, you have to deliver £1.

- The time zero cost is zero from Portfolio 1 and Portfolio 2.

- Then, it has to be that the forward price $F$ is

\[ F = \frac{1}{1 + r_{UK}} \times S_0 \times (1 + r_{US}). \]
Otherwise, there will be arbitrage.

Note, that the forward price is a function of

1. Current spot exchange rate $S_0$
2. UK lending and borrowing rate $r_{UK}$
3. US lending and borrowing rate $r_{US}$

It is not a function of

1. Your (and my) opinion about exchange rates
2. Your (and my) opinion about UK and US interest rates

*Forward spot parity in exchange rates is also called covered interest parity*
Forward Spot Parity — General Idea

Forward Spot parity is a valuation principle for forward contracts. Approximately,

\[
\text{Forward Price} = \text{Spot Price} + \text{“Cost of Carry”} - \text{Benefits of carry}
\]

Cost of Carry when the underlying is physical commodity = financing cost + warehousing cost + insurance + spoilage + transportation charges - convenience yield

Cost of Carry when the underlying is stock or stock index = financing cost - dividends received by the shareholder

Cost of Carry when the underlying is bond = financing cost - interest received by the bondholder
Example of carrying cost: Interest rate cost If we buy today instead of at expiry, we endure interest rate cost. In principle, we can save the money in the bank today and earn interests if we can buy it later.

Example of benefits of carry: Interest rate benefit If you buy pound (GBP) using dollar today instead of later, it costs you interest on dollar, but you can save the pound in the bank and make interest on pound.

Example of benefits of carry: Dividend benefit The idea is similar to the interest earned on the pound.

Idea: buying forward is equivalent to buying now and storing / carrying the underlying
Question: Consider a forward contract on the IBM stock. Assume that the stock is currently traded at $100 per share. IBM pays quarterly dividend of $1 per share in the next 5 years (no change). The continuously compounding interest rate is flat at 5% (across all maturities). Compute the forward price of the IBM stock at one year maturity.
For IBM stock, the cost is interest cost, and the benefit is dividends.

\[ F_1 = 100 \times e^{0.05 \times 1} \quad \text{(Cost of borrowing spot price)} \]
\[ - 1 \times e^{0.05 \times 3/4} \quad \text{(Benefit of investing the dividend received a quarter from now)} \]
\[ - 1 \times e^{0.05 \times 2/4} \quad \text{(Benefit of investing the dividend received 2 quarters from now)} \]
\[ - 1 \times e^{0.05 \times 1/4} \quad \text{(Benefit of investing the dividend received 3 quarters from now)} \]
\[ - 1 \times e^{0.05 \times 0/4} \quad \text{(Benefit of investing the dividend received 4 quarters from now)} \]
\[ = $101.05 \]
Stock Index Forward-Spot Parity

The forward price is \( F = S_0 \times (1 + r_f)^T - D \) where

1. \( S_0 \) is the value of the stock index now
2. \( r_f \) is the annualized risk free rate
3. \( D \) is the total cash value of the dividend until time \( T \)
4. \( T \) is the maturity date of the forward contract