Math 141 Sample Final Exam Problems

The final exam will cover Chapters 2, 3, 4, and 5, except sections 2.4, 4.6, and 4.8. This collection of sample questions is longer than the final, which will have approximately 10 questions.

1. Evaluate the derivatives of the following functions:

   (a) $f(x) = \sin\sqrt{x^2 + x + 1} \cdot \cos\sqrt[3]{x}$

   (b) $f(x) = \dfrac{x^2 + 1}{\cos^3(3x + 1)}$

   (c) $f(x) = x^3\tan(3x^2) + \sec(1/x)$

   (d) $f(x) = \displaystyle\int_{-3x}^{\sin x} \frac{1}{1 + t^2}\, dt$

2. Find the equation of the tangent line to the curve $y = f(x)$ at the point $(1, 0)$ if $xy^2 + 5y = x^3 - \cos(xy)$.

3. Determine the following limits:

   (a) $\displaystyle\lim_{x\to -2} \frac{x^3 + 8}{x^2 + x - 2}$

   (b) $\displaystyle\lim_{x\to 0} \frac{\sin^2(8x)}{\sin(8x^2)}$

   (c) $\displaystyle\lim_{x\to\infty} \sqrt{x^2 + 100x} - \sqrt{x^2 + 50x}$

   (d) $\displaystyle\lim_{x\to\infty} \frac{3 + \sin(7x)}{x}$

4. A trough is 10 feet long and its ends have the shape of isosceles triangles that are 3 feet across at the top and have a height of 1 foot. If the trough is being filled with water at a rate of 12 ft^3/min, how fast is the water rising when the water is 6 inches deep?

5. Find the area of the largest rectangle with sides parallel to the coordinate axes which can be inscribed in the ellipse $x^2/4 + y^2/9 = 1$.

6. Consider the function $f(x) = \dfrac{3x}{x^2 - 4}$. Then $f'(x) = \dfrac{-3(x^2 + 4)}{(x^2 - 4)^2}$ and $f''(x) = \dfrac{6x(x^2 + 12)}{(x^2 - 4)^3}$. Find all vertical and horizontal asymptotes. Find all local maxima and minima. Describe the intervals where the function is concave up and down. Describe the intervals where the function is increasing and decreasing. Sketch the graph.

7. State the definition of the derivative. State the definition of continuity.

8. Use the definition of the derivative and the identity $\sin(A + B) = \sin A\cos B + \cos A\sin B$ to find the derivative of $f(x) = \sin 5x$.

9. Show that the equation $\dfrac{x^2(x - 1)}{4 + 5x^2} = 1$ has at least one real solution.

10. Let $f(x) = x^4 + Ax^2 + x$, where A is a constant. Are there values of A for which f has inflection points at both $x = 0$ and $x = 1$?

11. Perform the required integrations:

(a) $\int 6\sec^2 5x\,dx$

(b) $\int \frac{\cos x}{\sqrt{1+2\sin x}}\,dx$

(c) $\int \frac{\cos\sqrt{x}}{5\sqrt{x}}\,dx$

(d) $\int_0^1 x^5\sqrt[7]{x^3+1}\,dx$

12. Consider the area under the curve $y = x + x^3$ between $x = 0$ and $x = 2$.

(a) Estimate the area using 4 subintervals and a right endpoint approximation.

(b) Find the exact area by setting up a Riemann sum and taking an appropriate limit. You may need the formulas $\sum_{i=1}^{n} i = \frac{1}{2}n(n+1), \sum_{i=1}^{n} i^2 = \frac{1}{6}n(n+1)(2n+1), \sum_{i=1}^{n} i^3 = \frac{1}{4}n^2(n+1)^2$.

13. "*If $f(x)$ is continuous at a point, then it is differentiable there.*"
"*If $f(x)$ is differentiable at a point, then it is continuous there.*"
One of these statements is true, and one is false.

(a) Which one is false?

(b) Give an example of a function that demonstrates the statement is not true.

14. Derive the iteration formula for Newton's method by the following two steps:

(Step 1) Write the equation for the tangent line to $f(x)$ at $x = x_n$ in terms of x_n, $f(x_n)$, and $f'(x_n)$.

(Step 2) Find the value of x where that tangent line intersects the x-axis. You may assume $f'(x_n) \neq 0$.

15. Use differentials to estimate the amount of paint needed to apply a coat of paint 0.05 cm thick to a hemispherical dome with radius 50 m.

16. Let

$$f(x) = \begin{cases} 2x - x^2 & \text{if } x < 0 \\ 3 - x & \text{if } 0 \le x < 3 \\ (x-3)^2 & \text{if } x > 3 \end{cases}$$

(a) Evaluate each limit, if it exists.
(i) $\lim_{x\to 0^+} f(x)$ (ii) $\lim_{x\to 0^-} f(x)$ (iii) $\lim_{x\to 0} f(x)$ (iv) $\lim_{x\to 3^-} f(x)$ (v) $\lim_{x\to 3^+} f(x)$ (vi) $\lim_{x\to 3} f(x)$

(b) Where is f discontinuous?

17. The velocity of a particle as a function of time is given by $v(t) = 3t^2 + 5t + 7 + 9\sin t$.

(a) Find the acceleration $a(t)$.

(b) Find the position $s(t)$, given $s(0) = 0$.