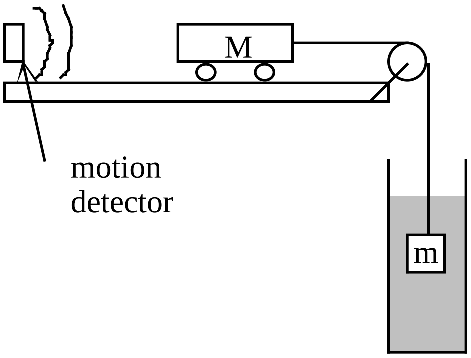
ONE-DIMENSIONAL SYSTEMS: Terminal Velocity

This experiment is a continuation of our study of one-dimensional systems. All of these systems are governed by an equation of the form $\dot{x} = f(x)$, where x is the dynamical variable. For this experiment x is the velocity of a moving cart.

Objectives:

1. Acquaintance with *Data Logger* software
2. Graphical Analysis:
  description of system behavior from graph
  identification of fixed points from plot of time dependence
  identification of fixed points from phase diagrams
3. Explore the fixed points:
  dependence on system parameters
  independence of initial conditions
  stability of fixed points

This experiment demonstrates a mechanical first-order system with the dynamical variable being the velocity. The setup is as shown in the figure.



A motion detector monitors the cart attached to a mass by a string passing over a pulley. The hanging mass is free to fall vertically in a column of liquid (detergent). One can start with different initial velocities in both the positive or the negative directions and observe the system reach the same terminal velocity.

The equation representing the system can be found by considering the forces acting on the hanging mass m:

$$ma = mg - Ma - f(v) \qquad \text{(Eq. 1)}$$

where a is the acceleration ($a = dv/dt = \dot{v}$), v is the velocity of the system, mg is the gravitational force on m, M is the mass of the cart, and the term Ma is due to the tension on the string, $f(v)$ is the drag force and can be linear or quadratic depending on the fluid, range of speeds, and the geometry. According to the theory $f(v)$ is linear ($f(v) \quad v$) for very low speeds and is quadratic ($f(v) \quad v^2$) at higher speeds. Equation 1 results in a dynamical equation of the form

$$\frac{dv}{dt} = \dot{v} = \frac{m}{m+M} g - \frac{1}{m+M} f(v) \qquad \text{(Eq. 2)}$$

In reality there is also the friction between the cart and the track, but we will assume that this is very small compared to the drag force on the falling mass.

Actions:

Set up the cart so that the hanging mass is immersed in the fluid at the top of the tube, with the string passing over the pulley and the motion detector at least 50 cm away from the cart. Align the fluid so that the mass does not hit the sides of the container. Also place two 500 g masses on the cart.

1. Locate the position of the cart on the track when the mass is at the bottom of the tube and place a piece of tape on the track about 6 inches before that point (because of a slight delay in the response of the motion detector you need to hit the stop key before the mass hits the bottom).

2. Open up the *Mac Motion* software and select a velocity versus time graph on the screen. Set the data rate to 10 pts/sec and averaging to 9 points. Set the time axis to 5 seconds and try a few runs to choose a suitable scale for the velocity axis.

3. First start the motion detector and release the cart from rest just before you hear the clicking sound from the detector. Also stop the motion detector when the cart reaches the location marked by the tape. This takes some practice so try it a few times before you actually save the data.
Wipe off the detergent on the string with a paper towel when taking out the mass so that the track does not get sticky with dripping detergent.

4. Print the velocity versus time graph and attach to your report.

5. Plot and print the acceleration versus time graph by choosing acceleration from the vertical axis selection (rescale the axis as necessary).

6. Plot and print acceleration vs. velocity (phase plot) by selecting the velocity from the horizontal axis (note that you might have to rescale the axis for a good graph).

7. Repeat the above for a different initial condition by pushing the cart towards the detector initially. When doing this, save the previous data as data A and select data B for this part so you can see both on the screen for comparison. (NOTE: Make sure the mass stays in the fluid at all times; do not push the cart too fast; push the cart just before you start the detector; do a few trial runs to get good data).

Questions:

1- Where is the fixed point? Is it stable? How do you know? What would you change to change the fixed point?
2- What is the initial condition for this experiment? Does the location of the fixed point depend on its value? Explain how you know.
3- Is this a linear system ? Explain your answer.
4- Examine your phase plot and try to guess the form of $f(v)$; does it agree with the theory mentioned above? Explain.
5-Choose a system parameter, e.g., hanging mass, shape of the hanging mass, type of fluid etc. Consider varying this parameter, and discuss the changes in the graphs of velocity vs. time and acceleration vs. velocity due to this variation. Does varying the parameter change the location of the fixed point? Explain.

SUMMARY

Compare and contrast all of the one dimensional systems; cooling, RC circuit, titration, and terminal velocity. Do they all have fixed points? Are all the fixed points stable? Are they all governed by the same equation ? Are they all linear?