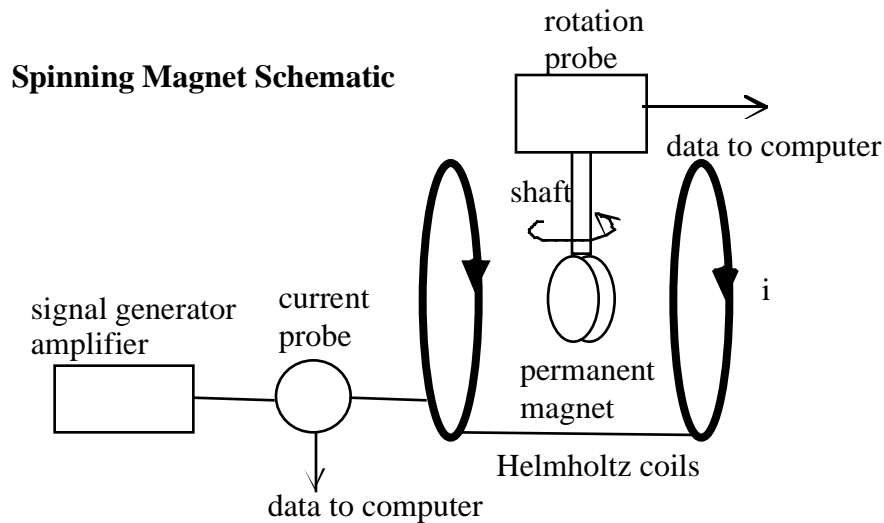


NONLINEAR SYSTEMS: Spinning Magnet

In this experiment we investigate techniques of display and analysis of data generated by the motion of a nonlinear mechanical system. This system shows an array of complex, though structured, behaviors which range from simple periodic to period doubled to chaotic. Our purpose is not to *explain* the observed behavior from first principles. These systems are too complex and sensitive to small simplifications in the model. Our purpose is to discover useful ways of plotting and analyzing the data so as to look for signatures of different types of behavior. The techniques we will investigate include *Fourier spectrum analysis*, *phase space trajectories*, and *Poincare sections*.

In these experiments, data is collected by means of electronic probes, which input directly into the computers in the lab. The software allows for immediate display of data in a variety of ways. We export the data to a numerical analysis program for detailed investigation.

Magnet in a time varying magnetic field



Introduction

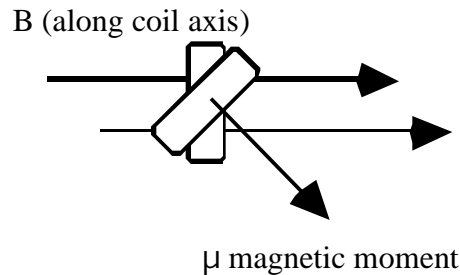
The schematic of this toy system is shown in the figure. The moving element is a small **permanent magnet** mounted in a region of space containing a magnetic field created by sending current through two large coils of wire. The external field created by this set of coils (called **Helmholtz coils**) is directed along the axis of symmetry, extremely spatially uniform, and proportional to the coil current. A sinusoidally time varying current is generated by the **amplified** output of a **signal generator**. The magnet tries to line up so that its magnetic moment (running from the South pole of the magnet to the North pole) aligns with the external magnetic field. If the external field is changing, however, the magnet is always spinning around to line up with it. The interplay of time variation of the external field, the moment of inertia of the spinning magnet, and frictional torques about the shaft drives the motion. The magnet spins on the shaft of an electronic **rotation probe** which measures rotation angle vs. time. This data is used to compute angular velocity. A **current probe** in series with the Helmholtz coils measures the current that creates the driving magnetic field.

A simple mathematical model of this system is given by the equation of motion relating the angular acceleration of the permanent magnet to the torques about the shaft axis acting on it.

$$I \ddot{\theta} = -b \dot{\theta} - \mu B(t) \sin(\theta)$$

where:

- I is the moment of inertia of the spinning magnet
- b is the linear coefficient of friction torque
- μ is the magnetic moment of the spinning magnet
- $B(t)$ is the external magnetic field, assumed to have a component perpendicular to the shaft directed along the axis of the Helmholtz coils
- $\theta(t)$ is the angle between the magnetic moment vector of the permanent magnet and the external magnetic field vector



Rotation Dynamics

In our experiment $B(t)$ is given by $B(t) = B_0 + B \cos(2\pi ft)$. The constant term, B_0 , arises from any DC bias in the driving current as well as from the Earth's magnetic field. The frequency of the time-varying driving field is f .

This equation is complex and nonlinear due to the explicit time dependence of $B(t)$ and due to the $\sin(\theta)$ term. The equation of motion can be written as a nonlinear system of equations as

$$\begin{aligned} \dot{\theta} &= -\frac{b}{I} \dot{\theta} - \frac{\mu B(t)}{I} \sin \theta \\ \dot{\omega} &= \ddot{\theta} \\ \dot{t} &= 1 \end{aligned}$$

The system, thus, has a three-dimensional phase space whose axes are the angle, angular velocity, and time (or any explicit function of time) variables. The angle and the time variables are periodic (measured, for example, on the interval $(0,1]$ as fraction of a revolution and fraction of a period). It is convenient to use the driving current, i , as the time variable since it is measured directly.

Remember that this equation is only a model. One of the most glaring simplifications in applying the model to this experiment is the functional form of the frictional torque. Although we can certainly run computer simulations of the model, it is not to be expected that the results of the simulation should be in detailed agreement with the actual data.

Activity

- Rotate the Helmholtz coils and the magnet/shaft/probe in its plastic cradle to align the shaft with the Earth's magnetic field using the dip needle. This eliminates the effect of the Earth's field from the dynamics.

- Start the **Rotary Motion** software and load the experiment file with **Open.../Phys 417-01/Rotary Motion/magzero**. On loading you will see angular velocity vs. time and current vs. time graphs. Add a phase plot (angular velocity vs. current) to observe the data.
- 1. Power up the probe interface and the frequency generator. Set the frequency to about 1 Hz. As you vary the current amplitude over the range 50 mA to 300 mA using the amplitude controls, you should be able to find (at least) the following types of behavior: periodic, period doubled, and chaotic motion. The type of motion depends on initial conditions, so try flicking the magnet to see if you can get the system to fall into different motions. You can see the data develop in real (or nearly real) time if you click on **Collect/All Graphs Live**. You should quickly figure out how to distinguish different types of behaviors from these different displays.
- 2. Find the driving frequency of the Helmholtz coils by using **Analyze/FFT** for the current vs. time data. Also look at the FFT of angular velocity vs. time to see any harmonics or period doubled behavior. You can "Copy" the data from the FFT window and then "paste" the data into an Excel file for plotting of the Fourier spectrum.
- When you are satisfied that you have a good data set which shows interesting motion, print out the graphs. (Make sure you give the graphs a descriptive title) and save the data as a text file for some further analysis. This is done from **File/Export Data As...**; be sure click on the "Text" button and save it in your folder with a name you will recognize. This will take a few minutes.

Data Analysis

By importing your text data files into the numerical analysis software (see MATLAB addendum), you can generate:

(Note that upon being loaded the angle data is automatically converted to values from zero to one measured in fractions of a revolution.)

- plots of data vs. time (this was already observed as data was taken).
- a plot of the motion as a trajectory in three dimensional phase space.
- plots of projections of the trajectory onto two dimensional surfaces in the phase space.
- Poincare sections of the trajectory
- a plot of the power spectra of the driver and the response.

Questions

- What do the phase plots tell you about the rotational motion of the magnet? Can you *read* a phase plot? What aspects of the phase plot provide signatures for distinguishing between periodic, period doubled, and chaotic motion? Compare and contrast features for different motions?
- What is a Poincare section? How is it related to a phase plot? What aspects of the Poincare section provide signatures for distinguishing between periodic, period doubled, and chaotic motion? Compare and contrast features for different motions?
- What aspects of the Fourier spectra provide signatures for distinguishing between periodic, period doubled, and chaotic motion? Compare and contrast features for different motions?

Other things to do

Here are some ideas of things to investigate if you have the time (or for your lab project):

- measure changes in dynamical behavior as you vary the orientation of the shaft so that the Earth's magnetic field plays a role.
- observe complex behavior of the magnet in oscillation (as opposed to rotation).
- observe behavior for other periodic input waveforms, including square wave and sawtooth.
- perturb the system and determine eigenvalues of the linearized map on the attractor.