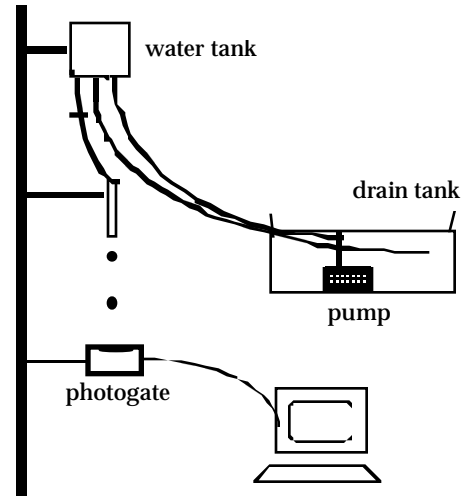


## DISCRETE SYSTEMS: The Dripping Faucet

Water dripping from a faucet, which is common in everyday life, displays many features of nonlinear systems including chaotic transitions, bifurcations, and multiple basins of attraction. Because of this complex behavior and its easy accessibility, investigating the dynamics of water dripping at different flow rates is a classic experiment. As the flow rate is varied the system can change from periodic to chaotic behavior. In this experiment we will measure the time interval between successive drops at different flow rates and learn techniques to quantify the behavior. Although it might not be of much practical use to quantify the behavior of water drops, these techniques can be applied to more complex problems such as cardiac rhythms, exchange rates, etc.

Our system consists of a water tank with a supply line and two outlet lines (one leading to a drain tank and the other to a thin pipette), a pump, a drain tank, and a photogate connected to the computer. The supply line is connected to a small pump which pumps the water back up to the water tank from the drain tank and the drain line drains the water tank when the water reaches a certain level. This maintains the water in the tank at a constant level providing a constant pressure. The hose leading to the pipette has a valve to adjust the flow rate. It can also be moved vertically relative to the tank for flow rate adjustment. The photogate is placed so drops fall in front of the beam which goes from arm to arm of the photogate. This starts a timer that stops when the next drop comes through and thus gives you the elapsed time between successive drops.



### To set up:

- Adjust the system so that water drops fall in front of the beam of the photogate and the pipette is vertical. Check that none of your tubes are pinched. Nonlinear systems are quite susceptible to small perturbations.

**Throughout the data run be very careful not to perturb the apparatus!**

- Start the photogate software (*ULI Timer*). Now let's look at the data as it is collected. Select Window/NewGraph. Properties of the graph (axes, etc.) are adjusted with Graph/Graph Options. Now click on the Start button and you should be able to see the interval between drops on the screen. Note that the photogate is continuously monitored until you choose Stop.
- You can now adjust the valve on the tube leading to the pipette so you have roughly 100-200 milliseconds between drops.

### Experiment:

Adjust the valve and/or move the pipette on the stand to change the flow rate and observe how the time intervals change. Notice that when the system is periodic, with one period, the dots form a steady horizontal line since the time interval between successive drops is always the same. When the drop rate starts fluctuating slightly there is some variation in the time intervals. When the system is in a "period doubled" state you see two different time intervals alternating, every other drop has the same time interval. Find flow rates that display the following behavior and collect and save data as explained below.

- 1-periodic
- 2-periodic with small fluctuations (~5-10%)

3-period doubled  
4-chaotic

### Collecting and saving data:

- When you have a flow rate that gives data exhibiting one of the above behaviors, stop the data collection. From the **File** menu choose **Save As...**, and save the data in your folder with a helpful name.
- Scale your axes so the data looks reasonable.
- Print out the graph of the data.

### Analyzing data:

Return map: One useful way of analyzing the data is by generating a "return map", which is a plot of successive time intervals,  $t_{n+1}$  vs.  $t_n$ . This is most easily accomplished in Excel.

1. Go to the "raw data" window in *ULI Timer* and click on the  $t$  column, it should become highlighted. Now copy the data by using command-C.
2. Open Excel, click in the second cell of column A, use command-V to paste the data here.
3. Now move to the first cell of column B, use command-V to copy it here as well.
4. Select the data (click and drag on the labels at the top of the columns A and B) and generate a plot using the Chart button. Adjust the axis scales so that they are the same (e.g. 0 to 300 on each axis).

Histogram: Another useful view comes from generating a histogram of all the time intervals between successive drops. You can do this in Excel using the Histogram tool.

For each type of behavior you should have the following graphs:

Do not connect the points by lines when you plot these graphs!

- 1-  $t$  versus  $t$ , this simple graph shows the time intervals as they occurred. It is generated by the *ULI Timer* software.
- 2- Return map (this is the analog of a Poincare section),  $t_{n+1}$  versus  $t_n$ .
- 3- Histogram of time intervals  $t_n$ .

### Questions:

- 1-What is the fundamental difference between this system and the previous ones we analyzed?
- 2-For the data set corresponding to each type of behavior explain how each of the above graphs quantifies the behavior.
- 3-What aspects of the histogram provide signatures for distinguishing between different types of behavior? Sketch the histogram you would expect to see for a period quadrupled system.