

## TWO-DIMENSIONAL SYSTEMS: Damped Simple Harmonic Oscillator

In the following experiment we will explore a two-dimensional system. These systems are governed by two equations of the form

$$\dot{x} = f(x, y), \quad \text{and} \quad \dot{y} = g(x, y)$$

where  $x$  and  $y$  represent the dynamical variables and  $f$  and  $g$  are functions of these variables. In some cases  $y$  represents the derivative of  $x$  with respect to time as we will see below. The systems that are explored in this lab are special types of two-dimensional systems, in that, they all execute oscillatory motion.

The mass-spring system is one of the simplest mechanical systems with oscillatory behavior. We have a vertical spring with a mass attached at one end monitored by a motion detector interfaced to a computer. The mass can be varied. The damping due to air resistance can be increased by attaching a piece of cardboard at the bottom of the mass to increase the surface area. The equation of motion for the mass is

$$m\ddot{x} = -kx - b\dot{x} + mg \quad (\text{Eq.1})$$

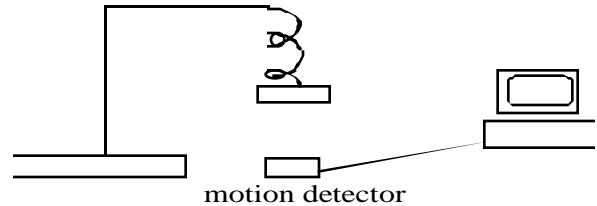
where  $x$  is the distance of the mass from equilibrium,  $k$  is the spring constant,  $-kx$  is the force due to the spring,  $b$  is the damping coefficient, and  $-b\dot{x}$  is the air resistance term. One can represent this as a two-dimensional system by defining  $x$  and  $v (= \dot{x})$  as the two dynamical variables. This results in the equations

$$\begin{aligned} \dot{x} &= v \\ \dot{v} &= -\frac{k}{m}x - \frac{b}{m}v + mg \end{aligned} \quad (\text{Eqns. 2})$$

Note that for this system we take the dynamical variables as position and velocity. Do see why the system is classified as linear?

### **Action:**

1. Hang the spring with the weight pan from the vertical support and place the motion detector below the spring. Make sure that the motion detector is at least 50 cm away from the mass at its lowest position.
2. Choose an amount of mass that results in steady oscillations and open up the *MacMotion* software. From the "Display" item on the menu choose "two graphs" and set one graph to  $x$  vs  $t$  and the other to  $v$  vs  $x$ .
3. Set the object in motion and observe the graphs as the mass oscillates.
4. Print out the graphs when you have recorded a number of oscillations and attach it to your report. Be sure you have some clean data.
5. Start with different initial conditions (i.e. start *MacMotion* at arbitrary times during the oscillation) and observe the graphs.



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**6. Change** the mass and repeat the steps above. (Save the data with the original trial as data A and record the new data as data B for direct comparison.)

7. Attach a sheet of cardboard at the bottom of the weight pan to increase the air resistance and repeat steps 1 to 5. If you attach a large enough piece you can get critical damping.

**Questions:**

**Responses to the following should be in your report.**

- 1- Is there a fixed point in this system? Does it depend on the mass? Does it depend on the amount of damping?
- 2- Does a change in the initial conditions change the fixed point?
- 3- How does changing mass affect the behavior of the system?
- 4- How does changing the damping affect the behavior of the system?