

ONE-DIMENSIONAL SYSTEMS: Cooling

This experiment, and others to follow, will explore the properties of one-dimensional systems. All of these systems are governed by an equation of the form $\dot{x} = f(x)$, where x is the dynamical variable and $f(x)$ is some function of x that depends on the specific system. In this course x is a general symbol used to represent a variety of things such as temperature, velocity, etc., and does not necessarily represent position.

Objectives:

1. Acquaintance with *Logger Pro* software
2. Graphical analysis:
 - description of system behavior from graph
 - identification of fixed points from time dependence
 - identification of fixed points from phase plots
3. Explore the fixed points and their:
 - dependence on system parameters
 - independence from initial conditions
 - stability

This experiment explores the thermal equilibration (cooling down or warming up) of a system (temperature probe) by monitoring its temperature as a function of time. We will use a temperature probe interfaced to a computer. The probe, which is initially hot or cold, is placed in water at room temperature and the probe's temperature is then monitored as a function of time. The *Logger Pro* software is used to record and process the data which is plotted on the screen in real time.

The dynamical variable for this system is the temperature T of the probe. An equation that models this process (Newton's law of "cooling") can be written as:

$$\dot{T} = k(T_a - T) \quad (\text{Eqn. 1})$$

where k is a positive constant (what are its units?) which depends on the conductivity, heat capacity, mass, thickness and surface area of the probe, the conductivity of the water, and where T_a is the ambient temperature (room temperature in this case). Note that we characterize this as a linear system since \dot{T} is a linear function of T .

Qualitative Expectations:

In general it can be useful to identify the qualitative (i.e. not numerical/quantitative) behavior of the model describing the system- in this case Eqn 1. This allows us to check the suitability of the model and/or tells us what qualitative features we can expect our data to exhibit. In this case let's see what the model predicts for the plot of temperature as a function of time (T versus t). We can do this without explicitly solving the equation.

- ◆ Responses for the following items should be included in your report. Answer the following questions separately for cooling and warming of the probe to room temperature. Use the answers to sketch the plot you expect for $T(t)$ versus t .

1. Where is the starting temperature $T(t=0)$ relative to T_a ?
2. What is the sign of the slope of $T(t)$ at this starting temperature?
3. Does the system have a fixed point- if so where? What value of the dynamical variable T ?
4. In going from the starting temperature to the fixed point, how does the slope change? Does it increase, decrease, change sign, etc?

Actions:

1. Begin by placing the probe in the hot water container so that most of the probe is immersed. Note that the instantaneous temperature of the probe can be monitored at the bottom of the screen. Once the probe temperature has come close to equilibrium with the hot water, *carefully* transfer it to the container of room temperature water, and immediately start *Logger Pro* (via the **Collect** button) to monitor the temperature of the probe.
2. Once the data collection has stopped, choose the *Store Latest Run* command from the **Data** item on the menu. This saves your data as *Run 1* and allows you to record another set of data points that appear on the same plot.
3. Repeat steps 1 through 3, but this time place the temperature probe in the ice-water mixture for a few minutes before you place it in the container of room temperature water.

Data analysis :

The *Logger Pro* software will automatically apply your actions to both data sets, so you will see both plots for each item below.

- Print the final T vs. time graph and attach it to your report.
- Plot a new graph of \dot{T} vs. T , i.e. a phase plot, print and attach to your report. You can do this by first taking the derivative of the temperature data and storing this in a new column. This is done via the *Data-> New Column -> Formula* command.
- Indicate the direction of the system trajectories on your phase plots with an arrow.

Questions:

- ◆ Responses for the following items should be included in your report.
1. Do your graphs of $T(t)$ agree qualitatively with the sketches of your expected plots?
 2. Explain the shape of the phase plots, circle the fixed point on your graph.
 3. How is this fixed point related to the constants in Eqn. 1?
 4. Describe a simple variation of this experiment that would change the location of the fixed point.
 5. What are the initial conditions for this experiment? Does the location of the fixed point depend on the initial conditions, explain your answer?
 6. Consider a similar experiment with a styrofoam layer covering the temperature probe. Discuss the changes in the graphs of T vs. t and the phase plot due to this variation. Does varying this system parameter change the location of the fixed point? Explain.

7. Imagine that on Planet Cubic, Newton's law of "cooling" obeys the equation

$$\dot{T} = h(T_a - T)^3 \quad (\text{Eqn. 2})$$

where h is a positive constant. Identify the qualitative behavior of the model and make separate sketches for the plots ($T(t)$ versus t) that you would expect for cooling and warming of the probe to room temperature. Sketch the system's phase plot.