# SUPERCRITICAL PITCHFORK BIFURCATION: Bead on a Hoop

In section 3.5 of Strogatz you studied the over-damped bead on a hoop. In this lab we will do an experiment to quantitatively verify the predictions of the model. You should review the derivation of the equation of motion (equations (1) and (2) in sect. 3.5) and the location of fixed points as a function of the parameter $\gamma = r\omega^2 / g$. **As part of your lab report also hand in a linear stability analysis of the fixed points (exercise 3.5.2).**

The experimental apparatus comprises a rotating hoop with a rail inside along which a ball bearing can roll. The first thing you'll notice is that the apparatus is *not* overdamped so that oscillations can occur. This is prohibited in one-dimensional systems. However, we will be able to use the analysis in Strogatz since we'll only be looking at the system in equilibrium.

The speed of the motor driving the hoop can be varied using the power supply. The frequency of rotation can be determined using either the strobe light or by simply touching your finger lightly against the axle of the hoop and counting the rotations for, say, 30 seconds.

Experiment

The radius of the hoop (i.e. from the hoop center to the center-of-mass of the ball bearing) is 6.7 +/- 0.2 cm.

* **Calculate the critical frequency at which the bifurcation occurs.**
* Experiment with the driving speed. You will note that most of the "action" takes place somewhere between 1.5 and 3 Hz. Does this make sense? Compare this observation with the graph in figure 3.5.6 in Strogatz.

To get a more quantitative feel for what is going on we'll now measure the angle, $\varphi$ (the fixed point), of the ball as a function of rotation frequency. To do this we'll videotape the hoop in rotation and then use the freeze-frame capability on the VCR to arrest the motion and allow us to measure the angle from the TV screen. The edge of the hoop has been marked with ticks corresponding to angle. Each tick = $5^0$.

I suggest that after experimenting a little to get a feel for the apparatus, raise the rotation rate to about 3 Hz and measure the frequency. So that you have a record of the frequency of rotation, write this on a piece of paper and place the paper in the field of view of the camera. Start the video camera and record the motion for about 5 seconds. Now reduce the frequency slightly, measure it, and record another 5 seconds of videotape, remembering to indicate the frequency by putting a new piece of paper in the frame. Repeat this procedure until the ball is at the bottom of the hoop and you have ~ 10 data points.

You can now replay the videotape and use the pause and frame advance to freeze the motion of the hoop so that you can get a measurement of the angle of the ball. As you do the experiment, try and get some feeling for the uncertainty associated with the measurements. For example, how accurately is the frequency measured? How well can you judge the angles?

To do:

Besides the bold-faced items above, also address the following points.

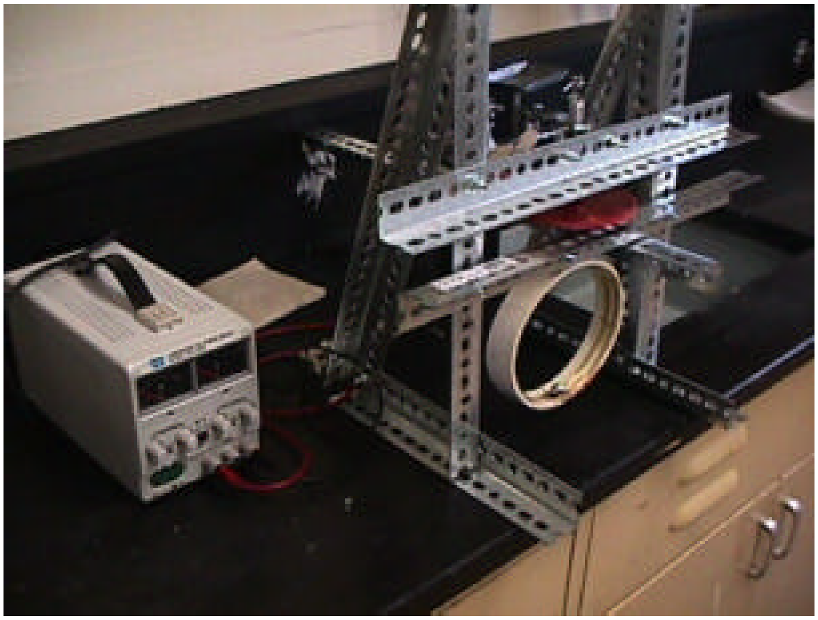
1) Plot your experimental $\varphi$ vs. $\gamma$ and show where the bifurcation seems to occur. Superimpose a plot of theoretical $\varphi$ vs. $\gamma$.

2) Since its probable that the bifurcation will not appear to occur exactly where expected, explain where you think the disagreement comes from. Consider the role of an imperfection parameter in this system.

3) Does the ball always go up the same side of the hoop or does it go up either side with equal probability? What "breaks the symmetry"? If the ball always goes up the same side, can we still use this apparatus to verify the model?

References:

1) If you would like to pursue this experiment in more detail, ask your instructor for a copy of "A mechanical analog of first- and second-order phase transitions" by G. Fletcher, *American Journal of Physics*, **65** (1), 74-81 (Jan 1997)



Bead on hoop apparatus