

SYLLABUS FOR THE WRITTEN EXAMINATION IN ALGEBRA

The examination covers basic properties of fundamental algebraic structures. The following is a (not necessarily exhaustive) list of topics from which questions are drawn.

1. LINEAR ALGEBRA

- Vector Spaces
subspaces, bases, dimension, direct sums
- Linear Transformations and Matrices
invertibility, matrix representation of a linear transformation, determinants, similarity, eigenvalues and eigenvectors, diagonalization, invariant subspaces, Jordan canonical form, minimal polynomial, characteristic polynomial, Cayley-Hamilton theorem
- Inner Product Spaces
adjoints, dual spaces, orthogonal projection, orthogonal complements, properties of Hermitian and orthogonal operators

2. GROUP THEORY

- Groups
basic properties of groups, subgroups, Lagrange's Theorem, normal subgroups, quotient groups, cyclic groups, permutation groups, simple groups (definition of; simplicity of A_n for $n \geq 5$), Cayley's Theorem, Fundamental Theorem of Finite Abelian Groups
- Homomorphisms
kernel, image, isomorphisms, isomorphism theorems, automorphisms

3. RING THEORY

- Rings
basic properties of rings, subrings, ideals, quotient rings, ring homomorphisms, isomorphism theorems, direct sums
- Integral Domains and Polynomial Rings
PID's, irreducible, prime, units, associates, UFD's, Euclidean domains, division algorithm, criteria for irreducibility
- Fields
characteristic, construction via quotient rings, fields as vector spaces, polynomial ring over a field

The material is found in a large number of texts, and is approached in a rather uniform fashion. Some texts that have been recently used are:

Linear Algebra

Axler, *Linear Algebra Done Right*

Friedberg, Insel and Spence, *Linear Algebra*

Lang, *Linear Algebra*

Group and Ring Theory

M. Artin, *Algebra*

Dummitt and Foote, *Abstract Algebra*

Fraleigh, *A First Course in Abstract Algebra*

Gallian, *Contemporary Abstract Algebra*

Herstein, *Abstract Algebra*