7. Shift in Time of a Series

In the factors developed previously involving a uniform series of amounts, the convention is that the first regular payment appears at the end of the first period. In an actual situation the first amount may appear at time 0, and no amount appears at the end, as illustrated in Figure 4. If the future worth of this series is sought, each of the terms in Table 2 could be multiplied by \((1 + i)\), with the result that the \(f/a\) factor for the shifted series is

\[
\frac{f}{a_{shift}} = (1+i) \left( \frac{(1+i)^n - 1}{i} \right)
\]

Figure 4. First Amount in Series Appearing at Time 0

8. Different Frequency of Series Amounts and Compounding

The factors involving uniform series amounts were developed on the basis of the same compounding and payment frequencies. A situation often arises where the periods are different, e.g., monthly compounding with annual payments. More frequent compounding than payments can be accommodated by determining an equivalent rate of interest applicable between the payment periods.

Example 11. Annual investments of $1,000 are to be made at a savings and loan institution for ten years beginning at the end of the first year. The institution compounds interest quarterly at a nominal annual interest rate of 4%. What is the expected value of the investment at the end of 10 years?

Solution: If an amount \(X\) starts drawing interest at the beginning of a year and interest is compounded quarterly at an annual rate of 4\%, the value \(Y\) at the end of the year is

\[
Y = X(1 + \frac{0.04}{4})^4 = X(1 + 0.0406)
\]

Therefore, the equivalent rate of interest due to the quarterly compounding is 4.06\%. This rate can then be used in the \(f/a\) factor to compute the future worth of the investment after 10 years

\[
\text{Future Worth} = (\text{Series Amount}) \left( \frac{a}{f}, i=0.0406, n=10 \right) = $12,040
\]
9. Evaluating Potential Investments
An important function of economic analyses in engineering enterprises is to evaluate potential investments. A commercial firm must develop a rate of return on its investment that is sufficient to pay corporation taxes and still leave enough to pay dividends on the stock that provides the investment capital. The evaluation can become very intricate, and only the basic investment situations will be discussed. This fundamental approach is, however, the starting point from which modifications and refinements can be made in more complicated situations.

The rate of return is an important concept in choosing between different investment alternatives. All the costs and incomes are considered when calculating the rate of return, which is the interest rate at which the income and the costs balance out. The following example demonstrates the calculation of the rate of return.

**Example 12.** Two plastic forming facilities, A and B, are suitable for a plastic recycling system. The following financial data are given for the two facilities:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost</td>
<td>$50,000</td>
<td>$80,000</td>
</tr>
<tr>
<td>Annual income</td>
<td>26,000</td>
<td>36,000</td>
</tr>
<tr>
<td>Annual maintenance and other costs</td>
<td>11,000</td>
<td>15,000</td>
</tr>
<tr>
<td>Salvage value</td>
<td>10,000</td>
<td>20,000</td>
</tr>
</tbody>
</table>

The economic life of the facilities is given as five years. Calculate the rates of return for the two cases.

**Solution:** The rate of return can be calculated using either the present worth or uniform annual amounts. In both cases, the rate of return is the interest rate which results when the costs are equal to the income. Using the present worth, the calculation for both cases is:

\[
(\text{Annual income} - \text{Annual costs})(\frac{P}{a}, i, n = 5) + \\
(\text{Salvage value})(\frac{P}{f}, i, n = 5) - \text{Initial cost} = 0
\]

Or, using uniform annual amounts, the calculation for both cases is:

\[
(\text{Annual income} - \text{Annual costs}) + \\
(\text{Salvage value})(\frac{a}{f}, i, n = 5) - (\text{Initial cost})(\frac{a}{p}, i, n = 5) = 0
\]

In both situations, using present worth or uniform annual amounts, the unknown in the equation is the nominal annual interest rate or the rate of return. These equations are algebraic, nonlinear equations and the rate of return can not be solved for explicitly. Therefore, a non-linear equation solver or iteration is required. The rate of return for the two cases is determined to be 19.05% and 15.16% for facilities A and B, respectively. Therefore, facility A is the better investment since it yields a higher rate of return.
10. Taxes
The money for operating the government and for financing services provided by the
government comes primarily from taxes. The inclusion of taxes in an economic analysis
is often important because in some cases taxes may be the factor deciding whether to
undertake a project or not. In certain other cases the introduction of tax considerations
may influence which of two alternatives will be more attractive economically.

In most parts of the United States, property taxes are levied by a sub-state taxing district
in order to pay for schools, city government and services, etc. Theoretically, the real
estate tax should decrease as the facility depreciates, resulting in lower real estate
taxes. Often, on investments such as buildings, the tax, as a dollar figure, never
decreases. It is therefore a common practice to plan for a common real estate tax when
making the investment analysis. The effect of the tax is to penalize a facility which has a
high taxable value.

Federal corporation income taxes on any but the smallest businesses amount to
approximately 50% of the profit. Since income tax is usually a much more significant
factor in the economic analysis than property tax, income tax will be discussed further.
An ingredient of income tax calculations is depreciation, explored in the next section.

11. Depreciation
Depreciation is an amount which is listed as an annual expense in the tax calculation to
allow for replacement of the facility at the end of its life. Several methods for computing
depreciation are allowed by the Internal Revenue Service, e.g., straight-line, sum-of-the-
year’s-digits, and double rate declining balance. The first two will be explained.

Straight-line depreciation consists simply of dividing the difference between the first cost
and salvage value of the facility by the number of years of tax life. The result is the
annual depreciation, which is a constant value. The tax life to be used is prescribed by
the Internal Revenue Service and may or may not be the same as the economic life
used in the economic analysis.

In the sum-of-the-year’s-digits (SYD) method, the depreciation for a given year is given
by the following formula

\[
\text{Depreciation ($)} = 2 \frac{N - t + 1}{N(N + 1)} (P - S)
\]

where  
N = tax life (years)  
t = year in question  
P = first cost ($)  
S = salvage value ($)

If the tax life is 10 years, for example, the depreciation is

\[
\text{First year} = 2 \frac{10}{110} (P - S)
\]
Second year = 2 \frac{9}{110} (P - S)

and continuing until the tenth year.

A comparison of the depreciation rates calculated by the straight-line method with those calculated by the SYD method shows that the SYD method permits greater depreciation in the early portion of the life. With the SYD method, the income tax that must be paid early in the life of the facility is less than with the straight-line method, although near the end of the life, the SYD tax is greater. The total depreciation amount and therefore the total tax paid over the tax life of the facility is the same by either method. The advantage of using the SYD method is that more of the tax is paid in later years, which is advantageous considering the time value of money.

The straight-line method of depreciation has an advantage, however, if it is likely that the tax rate will increase. If the rate jumps, it is better to have paid the low tax on a larger fraction of the investment.

12. Influence of Economic Tax on Investment
To see the effect of depreciation and federal income tax, consider the following example.

**Example 13.** Consider the following two investment alternatives:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost</td>
<td>$200,000</td>
<td>$270,000</td>
</tr>
<tr>
<td>Life</td>
<td>20 years</td>
<td>30 years</td>
</tr>
<tr>
<td>Salvage value</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Annual income</td>
<td>$60,000</td>
<td>$60,000</td>
</tr>
<tr>
<td>Annual operating expense</td>
<td>$14,000</td>
<td>$6,200</td>
</tr>
<tr>
<td>Real estate tax and insurance (5% of initial cost)</td>
<td>$10,000</td>
<td>$13,500</td>
</tr>
<tr>
<td>Interest</td>
<td>9%</td>
<td>9%</td>
</tr>
</tbody>
</table>

It is typical to calculate the profit and the taxes on an annual basis. The single expense shown above which is not an annual amount is the initial cost. The annual payments on the loan are calculated to be

\[ \text{Annual payment} = (\text{Initial cost})(\frac{a}{p}, \ i = 0.09, \ n = 20 \ or \ 30 \ years) \]

which yields $21,909 for investment A and $26,281 for investment B. Part of this payment is used to pay off the principal of the loan, while part is used to pay the interest on the loan. The interest on the loan is an expense which can be deducted in order to reduce the taxable income. To calculate the interest on the loan simply multiply the balance of the loan by the interest rate. For the first year, the interest paid is 9% of the initial cost which is $18,000 and $24,300 for investments A and B respectively. Therefore, for the first year of investment A, $18,000 of interest is paid while ($21,909 - $18,000) = $3,909 of the principal amount is paid. The reduction in the principal means that the unpaid balance during the second year is $196,091. The interest paid on the second year would be 9% of this amount. Similarly for investment B.
The depreciation per year is simply the initial cost divided by the number of years (since there is no salvage value), which yields $10,000 for investment A and $9,000 for investment B. This amount is constant over the tax life of the investment since straight-line depreciation is used. The resulting income tax on the two alternatives for the first year is shown in the following table.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depreciation</td>
<td>$10,000</td>
<td>$9,000</td>
</tr>
<tr>
<td>Interest (9% of unpaid balance)</td>
<td>$18,000</td>
<td>$24,300</td>
</tr>
<tr>
<td>Operating expense</td>
<td>$14,000</td>
<td>$6,200</td>
</tr>
<tr>
<td>Real estate tax and insurance (5% of initial cost)</td>
<td>$10,000</td>
<td>$13,500</td>
</tr>
<tr>
<td>Total expenses</td>
<td>$52,000</td>
<td>$53,000</td>
</tr>
<tr>
<td>Profit = income - expenses</td>
<td>$8,000</td>
<td>$7,000</td>
</tr>
<tr>
<td>Income tax (50% of profit)</td>
<td>$4,000</td>
<td>$3,500</td>
</tr>
</tbody>
</table>

13. Summary
There are several levels of economic analysis higher than that approached by this section. The complications in accounting, financing, and tax calculations involve sophistications beyond those presented here. The methods of investment analyses explained here are used repeatedly in engineering practice, and in most cases engineers are not required to go beyond these principles. These methods also are the base for extensions into more complex economic calculations.