Math 241 Sample Problems for Exam 1

**Question 1** Don't forget cylindrical and spherical coordinates and quadric surfaces.

**Question 2** Let \( f(x, y) = x \sin(y/x) \). Find the partial derivatives: \( \frac{\partial f}{\partial x} \), \( \frac{\partial f}{\partial y} \), \( \frac{\partial^2 f}{\partial y \partial x} \).

**Question 3** Find and sketch the domain of the function \( f(x, y) = \frac{5}{\sqrt{10 - 2y^2 - x^2}} \).

**Question 4** Let \( w = f(u, v) \) be a function whose derivatives of all orders exist. Suppose that \( \frac{\partial^2 f}{\partial u^2}(0, 2) = 0 \), \( \frac{\partial^2 f}{\partial u \partial v}(3, 0) = -3 \), \( \frac{\partial^2 f}{\partial u \partial v}(0, 2) = 2 \), \( \frac{\partial^2 f}{\partial v^2}(3, 0) = 3 \), \( \frac{\partial^2 f}{\partial v^2}(0, 2) = 1 \), \( \frac{\partial^2 f}{\partial v^2}(3, 0) = -1 \). If \( u = y + e^{2x} \) and \( v = xy \), what is the value of \( \frac{\partial^2 w}{\partial y^2} \) evaluated at the point \((x, y) = (0, 2)\).

**Question 5** Find the direction in which \( f(x, y) = x^2 + \cos xy \) increases most rapidly at the point \((1, \pi/2)\). What is the rate at which \( f \) changes in that direction? What is the equation of the tangent plane at the point \((1, \pi/2)\)?

**Question 6** Find the critical points of the function

\[
  f(x, y) = x^4 - x^2 y + \frac{3}{4} y^2 - 2y + 5
\]

and determine all relative maximum, relative minimum, and saddle points.

**Question 7** Evaluate the following double integral:

\[
  \int_0^2 \int_{y/2}^1 ye^x \, dx \, dy
\]

**Question 8** Find the volume of the solid in space which lies below the surface \( z = 3 + \cos y \) and above the region in the \( xy \)-plane bounded by the curves \( x = \pi \), \( y = 0 \), and \( y = 2x \) by evaluating an appropriate double integral.