Math 241 Sample Problems for Exam 2

Question 1 Evaluate the following double integral:
\[ \int_0^2 \int_{y/2}^1 ye^{x^3} \, dx \, dy \]

Question 2 Find the volume of the solid in space which lies below the surface \( z = 3 + \cos y \) and above the region in the \( xy \)-plane bounded by the curves \( x = \pi \), \( y = 0 \), and \( y = 2x \) by evaluating an appropriate double integral.

Question 3 Let \( R \) be the solid region bounded by the planes \( x = 0 \), \( y = 0 \), \( z = 2 \), and the paraboloid \( z = x^2 + y^2 \), in the first octant. Compute \( \iiint_R x \, dV \).

Question 4 Find the volume determined by \( z \leq 6 - x^2 - y^2 \) and \( z \geq \sqrt{x^2 + y^2} \).

Question 5 Convert the integral
\[
\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{x^4+y^4}} z^2 \, dz \, dx \, dy
\]

to spherical coordinates. Don't evaluate it.

Question 6 Evaluate \( \iint_R \exp\left( y - x y + x^2 \right) \, dA \) where \( R \) is the triangle with vertices \((0,0), (1,0), (0,1)\), by using the change of variables \( x = \frac{1}{2}(v-u), y = \frac{1}{2}(u+v) \).

Question 7
a) Evaluate the line integral \( \int_c x^2 y \, dx + xy^3 \, dy \), where \( c \) consists of the line segments from \((0,0)\) to \((3,3)\) and from \((3,3)\) to \((0,3)\).

b) Evaluate the line integral \( \int_c (e^y + y e^x) \, dx + (e^x + x e^y) \, dy \), where \( c \) is the part of the graph \( y = \ln 6x \) joining \((1/6,0)\) to \((1/2, \ln 3)\).

Question 8 Find the area of the surface that is part of the sphere \( x^2 + y^2 + z^2 = 4z \) that lies inside the paraboloid \( z = x^2 + y^2 \).