Math 241 Sample Problems for Exam 1

Question 1 Don’t forget cylindrical and spherical coordinates and quadric surfaces.

Question 2 Let \( f(x, y) = x \sin(y/x) \). Find the partial derivatives: \( \frac{\partial f}{\partial x} \), \( \frac{\partial f}{\partial y} \), \( \frac{\partial^2 f}{\partial y \partial x} \).

Question 3 Find and sketch the domain of the function \( f(x, y) = \frac{5}{\sqrt{10 - 2y^2 - x^2}} \).

Question 4 Evaluate the following limit:
\[
\lim_{(x,y) \to (1,2)} \frac{\sqrt{x+y} - \sqrt{3}}{x+y-3}
\]

Question 5 Show that the following limit does not exist as \( (x, y) \to (0, 0) \) by considering different paths of approach.
\[
\lim_{(x,y) \to (0,0)} \frac{x^2y + y^2}{x^4 + y^2}
\]

Question 6 Let \( w = f(u, v) \) be a function whose derivatives of all orders exist. Suppose that \( \frac{\partial^2 f}{\partial u^2}(0, 2) = 0 \), \( \frac{\partial^2 f}{\partial u \partial v}(0, 2) = 2 \), \( \frac{\partial^2 f}{\partial u^2}(3, 0) = 3 \), \( \frac{\partial^2 f}{\partial v^2}(0, 2) = 1 \), \( \frac{\partial^2 f}{\partial v^2}(3, 0) = -1 \). If \( u = y + e^{2x} \) and \( v = xy \), what is the value of \( \frac{\partial^2 w}{\partial y^2} \) evaluated at the point \( (x, y) = (0, 2) \).

Question 7 Find the direction in which \( f(x, y) = x^2 + \cos(xy) \) increases most rapidly at the point \((1, \pi/2)\). What is the rate at which \( f \) changes in that direction? What is the equation of the tangent plane at the point \((1, \pi/2)\)?

Question 8 Find the critical points of the function
\[
f(x, y) = x^4 - x^2y + \frac{3}{4}y^2 - 2y + 5
\]
and determine all relative maximum, relative minimum, and saddle points.