Math 241 Sample Problems for Exam 2

**Question 1** Use Lagrange multipliers to find the maximum and minimum values of \( f(x, y, z) = x - 2y + 5z \) on the sphere \( x^2 + y^2 + z^2 = 30 \).

**Question 2** Evaluate the following double integral:

\[
\int_0^2 \int_{y/2}^1 ye^x^3 \, dx \, dy
\]

**Question 3** Find the volume of the solid in space which lies below the surface \( z = 3 + \cos y \) and above the region in the \( xy \)-plane bounded by the curves \( x = \pi \), \( y = 0 \), and \( y = 2x \) by evaluating an appropriate double integral.

**Question 4** Let \( R \) be the solid region bounded by the planes \( x = 0 \), \( y = 0 \), \( z = 2 \), and the paraboloid \( z = x^2 + y^2 \), in the first octant. Compute \( \iiint_R x \, dV \).

**Question 5** Find the volume determined by \( z \leq 6 - x^2 - y^2 \) and \( z \geq \sqrt{x^2 + y^2} \).

**Question 6** Convert the integral

\[
\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} z^2 \, dz \, dx \, dy
\]

to spherical coordinates. Don’t evaluate it.

**Question 7** Evaluate \( \iint_R \exp \left( \frac{y - x}{y + x} \right) \, dA \) where \( R \) is the triangle with vertices \( (0, 0) \), \( (1, 0) \), \( (0, 1) \), by using the change of variables \( x = \frac{1}{2}(v - u) \), \( y = \frac{1}{2}(u + v) \).

**Question 8**

a) Evaluate the line integral \( \int_c x^2y \, dx + xy^3 \, dy \), where \( c \) consists of the line segments from \( (0, 0) \) to \( (3, 3) \) and from \( (3, 3) \) to \( (0, 3) \).

b) Evaluate the line integral \( \int_c (e^y + ye^x) \, dx + (e^x + xe^y) \, dy \), where \( c \) is the part of the graph \( y = \ln 6x \) joining \( (1/6, 0) \) to \( (1/2, \ln 3) \).

**Question 9** Find the area of the surface that is part of the sphere \( x^2 + y^2 + z^2 = 4z \) that lies inside the paraboloid \( z = x^2 + y^2 \).