Math 143 Sample Problems for Exam 2

**Question 1** Find the interval of convergence of the power series:

\[ \sum_{n=0}^{\infty} \frac{(-1)^n (3x + 1)^n}{4^{2n}(n+1)} \]

Don’t forget to check the endpoints!

**Question 2** Approximate the definite integral \( \int_{0}^{1} x \cos(3x^3) \, dx \) by using the first three non-zero terms of the Maclaurin series for \( f(x) = x \cos(3x^3) \). What is the maximum error in your approximation? You will need to know that \( \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \).

**Question 3** Find the Taylor series for \( f(x) = \cos x \) centered at the point \( x = \pi/4 \).

**Question 4** For the parametric curve \( x = e^t \cos t + \sin t, \quad y = e^t \cos t - \sin t \), find the equation of the tangent line at the point where \( t = \pi/4 \). Find the length of the curve from \( t = 0 \) to \( t = \pi/2 \). Set up, but do not evaluate an integral that represents the area of the surface of revolution gotten by rotating the curve from \( t = 0 \) to \( t = \pi/2 \) about the \( y \)-axis.

**Question 5** Find the area enclosed by the cardioid \( r = 2 + 2 \cos \theta \). Find the equation of the tangent line to the cardioid at the point when \( \theta = \pi/6 \).

**Question 6** Find the length of the spiral \( r = 3^{-\theta} \) from \( \theta = -\pi/2 \) to \( \theta = \pi/2 \).

**Question 7** Find the equation of the sphere whose diameter has endpoints \((1, 2, 3)\) and \((-1, 4, -6)\).

**Question 8** Something from sections 13.2.