Math 143 Sample Exam 1 Problems

**Question 1** For the parametric curve \( x = e^t (\cos t + \sin t), \ y = e^t (\cos t - \sin t) \), find the equation of the tangent line at the point where \( t = \pi/4 \). Find the length of the curve from \( t = 0 \) to \( t = \pi/2 \). Set up, but do not evaluate an integral that represents the area of the surface of revolution gotten by rotating the curve from \( t = 0 \) to \( t = \pi/2 \) about the \( y \)-axis.

**Question 2** Find the area enclosed by the cardioid \( r = 2 + 2 \cos \theta \). Find the equation of the tangent line to the cardioid at the point when \( \theta = \pi/6 \).

**Question 3** Find the length of the spiral \( r = 3^{-\theta} \) from \( \theta = -\pi/2 \) to \( \theta = \pi/2 \).

**Question 4** For each of the following sequences \( \{a_n\} \), decide whether it converges or diverges and circle the appropriate word. If the sequence converges, compute the limit of the sequence and write the limit of the sequence in the blank. (Show all work.)

a) \( a_n = \frac{\sqrt{2n^2 + 10n - 1}}{4 - 6n} \)  
Converges Diverges Limit = __________

b) \( a_n = \pi(3 - \cos(3/n)) \)  
Converges Diverges Limit = __________

c) \( a_n = \sin \left( \frac{\pi}{2} + (-1)^n \pi \right) \)  
Converges Diverges Limit = __________

d) \( a_n = \frac{(n + 2)!}{(n - 1)!} + \frac{(n + 1)!}{(n - 1)!} \)  
Converges Diverges Limit = __________

**Question 5** Compute the sum of the following infinite series:

a) \( \sum_{n=3}^{\infty} \frac{(-3)^{n-2}}{7^{n+1}} \)

b) \( \sum_{n=2}^{\infty} \frac{4}{n(n + 2)} \)