Question 1 Evaluate the derivatives of the following functions:

a) \( f(x) = \sin \sqrt{x^2 + x + 1} \cdot \cos \sqrt{x} \)

b) \( f(x) = \frac{\ln(x^2 + 1)}{\cos^3(3x + 1)} \)

c) \( f(x) = e^{x^3} \tan(3x^2) + \sec(1/x) \)

d) \( f(x) = \int_{-3x}^{\sin x} \frac{1}{1+t^2} \, dt \)

Question 2 Find the equation of the tangent line to the curve \( y = f(x) \) at the point \((1,0)\) if \( xy^2 + \ln(x + y) = x^3 - \cos(xy) \)

Question 3 Determine the following limits:

a) \( \lim_{x \to -2} \frac{x^3 + 8}{x^2 + x - 2} \)

b) \( \lim_{x \to 0} \frac{\sin^2 8x}{\sin(8x^2)} \)

c) \( \lim_{x \to -\infty} \sqrt{x^2 + 100x} - \sqrt{x^2 + 50x} \)

d) \( \lim_{x \to \infty} \frac{3 + \sin(e^x)}{x} \)

Question 4 Find the area of the largest rectangle with sides parallel to the coordinate axes which can be inscribed in the ellipse \( \frac{x^2}{4} + \frac{y^2}{9} = 1 \).

Question 5 Consider the function \( f(x) = \frac{3x}{x^2 - 4} \). Then \( f'(x) = \frac{-3(x^2 + 4)}{(x^2 - 4)^2} \) and \( f''(x) = \frac{6x(x^2 + 12)}{(x^2 - 4)^3} \). Find all vertical and horizontal asymptotes. Find all relative maxima and minima. Describe the intervals where the function is concave up and down. Describe the intervals where the function is increasing and decreasing. Sketch graph.

Question 6 Use the definition of derivative and the identity \( \sin(A + B) = \sin A \cos B + \cos A \sin B \) to find the derivative of \( f(x) = \sin 5x \).

Question 7 Show that the equation \( \frac{x^2(x - 1)}{4 + 5x^2} = 1 \) has at least one real solution

Question 8 Let \( f(x) = x^4 + Ax^2 + x \), where \( A \) is a constant. Are there values of \( A \) for which \( f \) has inflection points at both \( x = 0 \) and \( x = 1 \)?

Question 9 Perform the required integrations:

a) \( \int 6 \sec 5x \, dx \)

b) \( \int \frac{\cos x}{\sqrt{1 + \sin x}} \, dx \)

c) \( \int \frac{1 - e^{2x}}{e^{2x}} \, dx \)

d) \( \int_1^2 \frac{e^{1/x}}{x^2} \, dx \)
Question 10

a) Use a right endpoint approximation to estimate the area under the curve
   \( y = x + x^3 \) over the interval \([0, 2]\) using 4 subintervals.

b) Find the exact area under the curve above by setting up a Riemann sum
   and taking an appropriate limit. You may need the formulas:
   \[ \sum_{i=1}^{n} i = \frac{1}{2} n(n + 1), \]
   \[ \sum_{i=1}^{n} i^2 = \frac{1}{6} n(n + 1)(2n + 1), \]
   \[ \sum_{i=1}^{n} i^3 = \frac{1}{4} n^2(n + 1)^2 \]

The topics covered on the final are Chapter 1, Chapter 2, Chapter 3
   (except for sections 3.7 & 3.8), Chapter 4 (except 4.2 & 4.9), 5.1,
   6.1–6.4.